Name KEY

Math 152, Exam #3, Fall 2011

Instructions: Show all work. Complete all parts of each problem. Some problems will ask you to complete the entire problem (give exact answers and not decimals), and some will just ask you to set them up.

1. Use the attached table of integrals to integrate. State the number of the integral you use, and any constants or substitutions you need to identify for the problem. Be sure to give the answer. No need to simplify. Only two are required, but you can do all three.

   a. $\int xe^x \sin x dx$

   $\frac{1}{2}e^x(\cos x - x\cos x + x\sin x) + C$ #108

   b. $\int \sqrt{x(4x+3)}dx$

   #27

   $a=4$

   $b=3$

   $\frac{1}{4(4)^{3/2}}\left[(8x+3)\sqrt{4x(4x+3)} - 9\ln\left|4\sqrt{x}+\sqrt{4(4x+3)}\right|\right]$

   $+C$

   c. $\int \sin^2 2x\cos^2 5x dx$

   #76

   $a=2$

   $b=5$

   $\frac{x}{4} - \frac{\sin 4x}{16} - \frac{\sin(-6x)}{-48} + \frac{\sin 10x}{40} - \frac{\sin 14x}{112}$

   $a-b=-3$

   $a+b=7$

   $+C$

Integrals with Trigonometric Functions

$$\int \sin ax dx = -\frac{1}{a}\cos ax \tag{63}$$

$$\int \sin^2 ax dx = \frac{x}{2} - \frac{\sin 2ax}{4a} \tag{64}$$

$$\int \sin^n ax dx = -\frac{1}{a}\cos ax \;{}_2F_1\left[\frac{1}{2}, \frac{1-n}{2}, \frac{3}{2}, \cos^2 ax\right] \tag{65}$$

$$\int \sin^3 ax dx = -\frac{3\cos ax}{4a} + \frac{\cos 3ax}{12a} \tag{66}$$

$$\int \cos ax dx = \frac{1}{a}\sin ax \tag{67}$$

$$\int \cos^2 ax dx = \frac{x}{2} + \frac{\sin 2ax}{4a} \tag{68}$$

$$\int \cos^p ax dx = -\frac{1}{a(1+p)}\cos^{1+p} ax \times {}_2F_1\left[\frac{1+p}{2}, \frac{1}{2}, \frac{3+p}{2}, \cos^2 ax\right] \tag{69}$$

$$\int \cos^3 ax dx = \frac{3\sin ax}{4a} + \frac{\sin 3ax}{12a} \tag{70}$$

$$\int \cos ax \sin bx dx = \frac{\cos[(a-b)x]}{2(a-b)} - \frac{\cos[(a+b)x]}{2(a+b)}, a \neq b \tag{71}$$

$$\int \sin^2 ax \cos bx dx = -\frac{\sin[(2a-b)x]}{4(2a-b)} + \frac{\sin bx}{2b} - \frac{\sin[(2a+b)x]}{4(2a+b)} \tag{72}$$

$$\int \sin^2 x \cos x dx = \frac{1}{3}\sin^3 x \tag{73}$$

$$\int \cos^2 ax \sin bx dx = \frac{\cos[(2a-b)x]}{4(2a-b)} - \frac{\cos bx}{2b} - \frac{\cos[(2a+b)x]}{4(2a+b)} \tag{74}$$

$$\int \cos^2 ax \sin ax dx = -\frac{1}{3a}\cos^3 ax \tag{75}$$

$$\int \sin^2 ax \cos^2 bx dx = \frac{x}{4} - \frac{\sin 2ax}{8a} - \frac{\sin[2(a-b)x]}{16(a-b)} + \frac{\sin 2bx}{8b} - \frac{\sin[2(a+b)x]}{16(a+b)} \tag{76}$$

$$\int \sin^2 ax \cos^2 ax dx = \frac{x}{8} - \frac{\sin 4ax}{32a} \tag{77}$$

$$\int \tan ax dx = -\frac{1}{a}\ln \cos ax \tag{78}$$

$$\int \tan^2 ax dx = -x + \frac{1}{a}\tan ax \tag{79}$$

$$\int \tan^n ax dx = \frac{\tan^{n+1} ax}{a(1+n)} \times {}_2F_1\left(\frac{n+1}{2}, 1, \frac{n+3}{2}, -\tan^2 ax\right) \tag{80}$$

$$\int \tan^3 ax dx = \frac{1}{a}\ln \cos ax + \frac{1}{2a}\sec^2 ax \tag{81}$$

$$\int \sec x dx = \ln|\sec x + \tan x| = 2\tanh^{-1}\left(\tan\frac{x}{2}\right) \tag{82}$$

$$\int \sec^2 ax dx = \frac{1}{a}\tan ax \tag{83}$$

$$\int \sec^3 x \,\mathrm{dx} = \frac{1}{2}\sec x \tan x + \frac{1}{2}\ln|\sec x + \tan x| \tag{84}$$

$$\int \sec x \tan x dx = \sec x \tag{85}$$

$$\int \sec^2 x \tan x dx = \frac{1}{2}\sec^2 x \tag{86}$$

$$\int \sec^n x \tan x dx = \frac{1}{n}\sec^n x, n \neq 0 \tag{87}$$

$$\int \csc x dx = \ln\left|\tan\frac{x}{2}\right| = \ln|\csc x - \cot x| + C \tag{88}$$

$$\int \csc^2 ax dx = -\frac{1}{a}\cot ax \tag{89}$$

$$\int \csc^3 x dx = -\frac{1}{2}\cot x \csc x + \frac{1}{2}\ln|\csc x - \cot x| \tag{90}$$

$$\int \csc^n x \cot x dx = -\frac{1}{n}\csc^n x, n \neq 0 \tag{91}$$

$$\int \sec x \csc x dx = \ln|\tan x| \tag{92}$$

Products of Trigonometric Functions and Monomials

$$\int x \cos x dx = \cos x + x\sin x \tag{93}$$

$$\int x \cos ax dx = \frac{1}{a^2}\cos ax + \frac{x}{a}\sin ax \tag{94}$$

$$\int x^2 \cos x dx = 2x\cos x + (x^2 - 2)\sin x \tag{95}$$

$$\int x^2 \cos ax dx = \frac{2x\cos ax}{a^2} + \frac{a^2x^2 - 2}{a^3}\sin ax \tag{96}$$

$$\int x^n cosx dx = -\frac{1}{2}(i)^{n+1}\left[\Gamma(n+1, -ix) + (-1)^n\Gamma(n+1, ix)\right] \tag{97}$$

$$\int x^n cosax dx = \frac{1}{2}(ia)^{1-n}\left[(-1)^n\Gamma(n+1, -iax) - \Gamma(n+1, ixa)\right] \tag{98}$$

$$\int x \sin x dx = -x\cos x + \sin x \tag{99}$$

$$\int x \sin ax dx = -\frac{x\cos ax}{a} + \frac{\sin ax}{a^2} \tag{100}$$

$$\int x^2 \sin x dx = (2 - x^2)\cos x + 2x\sin x \tag{101}$$

$$\int x^2 \sin ax dx = \frac{2 - a^2x^2}{a^3}\cos ax + \frac{2x\sin ax}{a^2} \tag{102}$$

$$\int x^n \sin x dx = -\frac{1}{2}(i)^n\left[\Gamma(n+1, -ix) - (-1)^n\Gamma(n+1, -ix)\right] \tag{103}$$

Products of Trigonometric Functions and Exponentials

$$\int e^x \sin x dx = \frac{1}{2}e^x(\sin x - \cos x) \tag{104}$$

$$\int e^{bx} \sin ax dx = \frac{1}{a^2+b^2}e^{bx}(b\sin ax - a\cos ax) \tag{105}$$

$$\int e^x \cos x dx = \frac{1}{2}e^x(\sin x + \cos x) \tag{106}$$

$$\int e^{bx} \cos ax dx = \frac{1}{a^2+b^2}e^{bx}(a\sin ax + b\cos ax) \tag{107}$$

$$\int xe^x \sin x dx = \frac{1}{2}e^x(\cos x - x\cos x + x\sin x) \tag{108}$$

$$\int xe^x \cos x dx = \frac{1}{2}e^x(x\cos x - \sin x + x\sin x) \tag{109}$$

Integrals of Hyperbolic Functions

$$\int \cosh ax dx = \frac{1}{a}\sinh ax \tag{110}$$

$$\int e^{ax}\cosh bx dx = \begin{cases} \dfrac{e^{ax}}{a^2-b^2}[a\cosh bx - b\sinh bx] & a \neq b \\ \dfrac{e^{2ax}}{4a} + \dfrac{x}{2} & a = b \end{cases} \tag{111}$$

$$\int \sinh ax dx = \frac{1}{a}\cosh ax \tag{112}$$

$$\int e^{ax}\sinh bx dx = \begin{cases} \dfrac{e^{ax}}{a^2-b^2}[-b\cosh bx + a\sinh bx] & a \neq b \\ \dfrac{e^{2ax}}{4a} - \dfrac{x}{2} & a = b \end{cases} \tag{113}$$

$$\int e^{ax}\tanh bx dx = \begin{cases} \dfrac{e^{(a+2b)x}}{(a+2b)}\,{}_2F_1\left[1+\dfrac{a}{2b}, 1, 2+\dfrac{a}{2b}, -e^{2bx}\right] - \dfrac{1}{a}e^{ax}\,{}_2F_1\left[\dfrac{a}{2b}, 1, 1E, -e^{2bx}\right] & a \neq b \\ \dfrac{e^{ax} - 2\tan^{-1}[e^{ax}]}{a} & a = b \end{cases} \tag{114}$$

$$\int \tanh ax \, dx = \frac{1}{a}\ln\cosh ax \tag{115}$$

$$\int \cos ax \cosh bx dx = \frac{1}{a^2+b^2}\left[a\sin ax \cosh bx + b\cos ax \sinh bx\right] \tag{116}$$

$$\int \cos ax \sinh bx dx = \frac{1}{a^2+b^2}\left[b\cos ax \cosh bx + a\sin ax \sinh bx\right] \tag{117}$$

$$\int \sin ax \cosh bx dx = \frac{1}{a^2+b^2}\left[-a\cos ax \cosh bx + b\sin ax \sinh bx\right] \tag{118}$$

$$\int \sin ax \sinh bx dx = \frac{1}{a^2+b^2}\left[b\cosh bx \sin ax - a\cos ax \sinh bx\right] \tag{119}$$

$$\int \sinh ax \cosh ax dx = \frac{1}{4a}\left[-2ax + \sinh 2ax\right] \tag{120}$$

$$\int \sinh ax \cosh bx dx = \frac{1}{b^2-a^2}\left[b\cosh bx \sinh ax - a\cosh ax \sinh bx\right] \tag{121}$$

Name KEY

Math 152, Exam #3, Fall 2011

Instructions: Show all work. Complete all parts of each problem. Some problems will ask you to complete the entire problem (give exact answers and not decimals), and some will just ask you to set them up.

1. Question #1 is on a separate page. (10 points)

2. For each of the following integrals, determine if the problem can be done by parts. If not, state the rule or technique you would use (including if you have to use algebra first; for substitution, state *u*). If it can be, give *u* and *dv*. Do not complete any integration. (5 points each)

   a. Example: $\int \ln x dx$ Can it be done by parts? YES NO

      If NO, what method would you use?

      If YES: u= lnx dv= dx

   b. Example: $\int x\sqrt{1-x^2}\,dx$ Can it be done by parts? YES NO

      If NO, what method would you use? Substitution u=1-x^2

      If YES: u= dv=

   c. $\int x^4\sqrt{1-x^2}\,dx$ Can it be done by parts? YES NO it can, but probably shouldn't be

      If NO, what method would you use? trig sub $u = \sin\theta$

      If YES: u= x^3 dv= $x\sqrt{1-x^2}\,dx$

   d. $\int x \arctan x dx$ Can it be done by parts? YES NO

      If NO, what method would you use?

      If YES: u= arctan x dv= x dx

      will need trig sub for 2nd step (or another by parts)

   e. $\int \frac{x^2+1}{x-2}dx$ Can it be done by parts? YES NO

      If NO, what method would you use? long division

      If YES: u= dv=

4. Set up the trigonometric integral with the appropriate substitutions. Make any trig identity replacements needed. Do not complete the integration. (7 points each)

   a. Example: $\int \csc^6\theta \cot^4\theta d\theta$ u=cotθ, du=-csc²θ

$$\int \csc^2\theta\left(\csc^2\theta\right)^2 \cot^4\theta d\theta = \int \csc^2\theta\left(1+\cot^2\theta\right)^2 \cot^4\theta d\theta = -\int (1+u^2)^2 u^4 du$$

   b. $\int \cos^8\varphi \sin 7\varphi d\varphi$

$\sin^7\varphi$

$\cos\varphi = u$

$du = -\sin\varphi$

$\boxed{-\int u^8 (1-u^2)^3 du}$

$\sin^6\varphi = (1-\cos^2\varphi)^3$

   c. $\int \tan^9\psi \sec^{24}\psi d\psi$

$\sec\psi = u$

$\sec\psi \tan\psi = du$

$\tan^8\psi = (\sec^2\psi - 1)^4$

$\int (u^2-1)^4 u^{23} du$

OR

$\int u^9 (u^2+1)^{11} du$

or $u = \tan\psi$

$du = \sec^2\psi$

$\sec^{22}\psi = (\tan^2\psi + 1)^{11}$

5. Integrate. (10 points each)

   a. $\int \cos^3 t \sin^3 t dt$

$u = \cos t$

$du = -\sin t\, dt$

$\int \cos^3 t (1-\cos^2 t) \sin t\, dt$

$-\int u^3 - u^5\, dt$

$\boxed{-\frac{1}{4}\cos^4 t + \frac{1}{6}\cos^6 t + C}$

or

$u = \sin t$

$du = \cos t$

OR

$\int \cos t\,(1-\sin^2 t)\sin^3 t\, dt$

$\int u^3 - u^5\, dt$

$\boxed{\frac{1}{4}\sin^4 t - \frac{1}{6}\sin^6 t + C}$

7. A model for the ability M of a child to memorize, measured on a scale from 0 to 10, is given by $M = 1 + 1.6t\ln t, 0 \le t \le 4$ where t is the child's age in years. Find the average value of this model a) between the child's first and second birthdays, b) between the child's third and fourth birthdays. (10 points)

a) $\frac{1}{1}\int_1^2 1 + 1.6t\ln t\,dt$ $u = \ln t$ $dv = t$ $du = \frac{1}{t}dt$ $v = \frac{1}{2}t^2$

$t + 1.6\left[\frac{1}{2}t^2\ln t - \int \frac{1}{2}\cancel{t^2}\cdot\frac{1}{\cancel{t}}\,dt\right]$

$t + 1.6\left(\frac{1}{2}t^2\ln t - \frac{1}{4}t^2\right)\Big|_1^2 = 2 + .8(2)^2\ln 2 - \frac{1.6}{4}(2)^2 - 1 - 0 + \frac{1.6}{4}(1) =$

$\boxed{2.018}$

b) $\frac{1}{1}\int_3^4 1 + 1.6t\ln t\,dt =$

$t + 1.6\left(\frac{1}{2}t^2\ln t - \frac{1}{4}t^2\right)\Big|_3^4 = 4 + .8(4)^2\ln 4 - \frac{1.6}{4}(4)^2 - 3 - \frac{1.6}{2}\ln 3 + \frac{1.6}{4}(3)^2 =$

$\boxed{8.0346}$

8. Set up, but do not integrate. (10 points)

$$\int \frac{x^3 + 5x^2 - 8x - 24}{(x^2+1)^2(x-3)^3(x+4)(x-1)}dx$$

$$\int \frac{Ax+B}{x^2+1} + \frac{Cx+D}{(x^2+1)^2} + \frac{E}{x-3} + \frac{F}{(x-3)^2} + \frac{G}{(x-3)^3} + \frac{H}{(x+4)} + \frac{I}{x-1}\,dx$$

10. Find the limit. (8 points each)

a. $\lim_{x\to 0}\frac{\arcsin x}{x} = \frac{0}{0}$

$$\frac{\frac{1}{\sqrt{1-x^2}}}{1} = \frac{1}{1} = \boxed{1}$$

b. $\lim_{x\to 0}(\sin x)^x = L$ $\quad 0^0$ indef.

$\ln(\sin x)^x = \ln L$

$x\ln\sin x = \ln L \qquad 0\cdot -\infty$

$$\frac{-\infty}{\infty} \quad \frac{\ln(\sin x)}{\frac{1}{x}} \Rightarrow \frac{\frac{1}{\sin x}\cdot\cos x}{-\frac{1}{x^2}} = \frac{\cot x}{-x^{-2}} = -\frac{x^2}{\tan x}$$

$$= \frac{0}{0} \Rightarrow \frac{-2x}{\sec^2 x} = \frac{-2(0)}{1^2} = \boxed{0}$$

$\ln L = 0$

$L = e^0 = \boxed{1}$

#6. by parts

$\frac{x^5}{\sqrt{8-x^2}}$ $\quad u = x^4 \quad dv = x(8-x^2)^{-1/2}$

$du = 4x^3 \quad v = \cancel{\tfrac{1}{2}} - (8-x^2)^{1/2}$

$-x^4(8-x^2)^{1/2} + \int 4x^3(8-x^2)^{1/2}$

$u = x^2 \quad dv = x(8-x^2)^{1/2}$

$du = 2x \quad v = -\frac{\cancel{2}}{3}(8-x^2)^{3/2}$

$-x^4(8-x^2)^{1/2} - \frac{4}{3}x^2(8-x^2)^{3/2} + \int \frac{2}{3}x(8-x^2)^{3/2}\,dx$

$-x^4(8-x^2)^{1/2} - \frac{4}{3}x^2(8-x^2)^{3/2} - \frac{1}{3}\cdot\frac{2}{5}(8-x^2)^{5/2} + C$

$-x^4(8-x^2)^{1/2} - \frac{4}{3}x^2(8-x^2)^{3/2} - \frac{2}{15}(8-x^2)^{5/2} + C$

#3 by change of variable

$u = \sqrt[3]{2x+1} \quad u^3 = 2x+1 \quad \frac{u^3-1}{2} = x$

$3u^2\,du = 2\,dx$

$\int\left(\frac{u^3-1}{2}\right)^2 \cdot \frac{1}{\cancel{u}} \quad \frac{3}{2}u^{\cancel{2}}\,du =$

$\frac{3}{16}\int(u^6-2u^3+1)u\,du =$

$\frac{3}{16}\int u^7-2u^4+u\,du =$

$\frac{3}{16}\left[\frac{1}{8}u^8 - \frac{2}{5}u^5 + \frac{1}{2}u^2\right]+C$

$\frac{3}{16}\left[\frac{1}{8}(2x+1)^{8/3} - \frac{2}{5}(2x+1)^{5/3} + \frac{1}{2}(2x+1)^{2/3}\right]+C$

$\frac{3}{128}(2x+1)^{8/3} - \frac{3}{40}(2x+1)^{5/3} + \frac{3}{32}(2x+1)^{2/3} + C$