Section 4.1 Solving Systems of Linear Equation in Two Variables
Math 103 Course Outline Unit I
Objective: Solve a system of equations graphically.

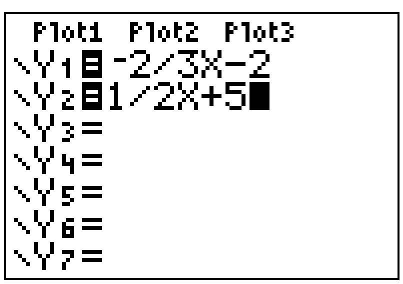
New calculator features used in this lesson:

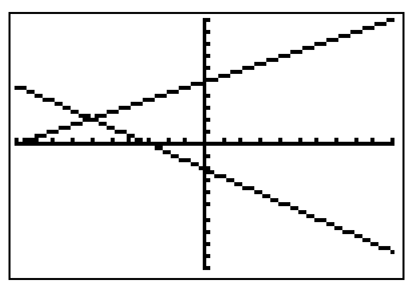
- 2ND TRACE (CALC) feature "intersect"

> <u>INSTRUCTOR NOTE</u>
>
> ❖ ***After*** students have mastered the concept of solving a system of linear equations using "paper and pencil" methods, demonstrate how to graphically verify the solutions found algebraically.
>
> ❖ Applied problems which are modeled by systems of linear equations may be better suited for solving graphically without first solving algebraically.

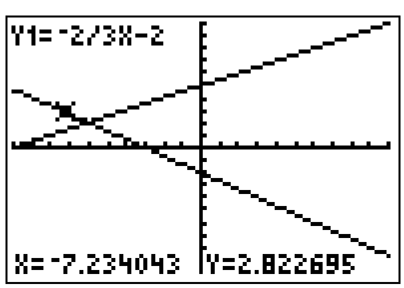
<u>Example 1:</u> Solve the system $\begin{cases} y = -\frac{2}{3}x - 2 \\ x - 2y = -10 \end{cases}$ graphically.

<u>Solution:</u> Turn off or clear any equations in the Y= editor. Then solve each equation for *y* and enter them as Y1 and Y2: Y1 $= -\frac{2}{3}x - 2$ and Y2$=\frac{1}{2}x + 5$. Use the Standard window for the initial graph. NOTE: the second equation could also be entered in the calculator as $y = \frac{x+10}{2}$, but caution students to enter the expression correctly, with parentheses around the numerator!

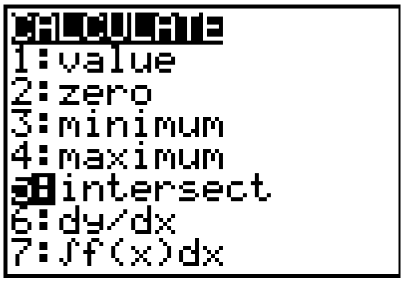




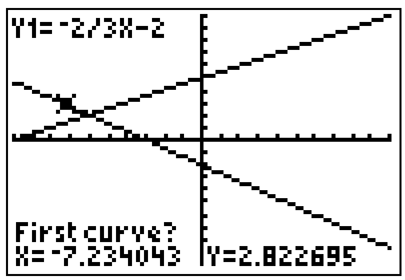
Use the TRACE key to move the cursor close to the point of intersection. (Notice that when you are in TRACE, the equation of Y1 appears in the upper left corner of the screen. Hit the down arrow button, and the equation of Y2 appears in the upper left corner.)



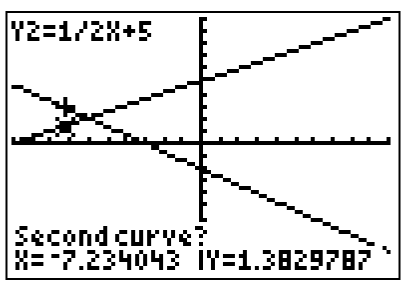
Step 1: The “intersect” feature is in the CALC menu. To access the “intersect” feature, press 2ND TRACE. This brings up the CALC menu. Scroll down to 5: intersect, and press ENTER. (Or simply press the number 5.)



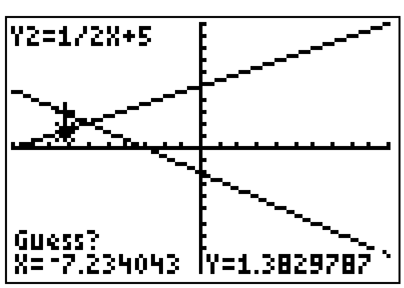
Step 2: Notice the prompt “First curve?”. The calculator is prompting you to place your cursor on the first graph (Y1), if you haven’t already done so. See the next screen shot.

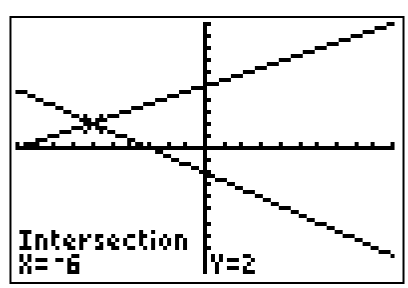


Step 3: Press ENTER and notice the second prompt, “Second curve?”.



Step 4: Press ENTER. The last prompt is “Guess?” If you want to move your cursor so that it is closer to the point of intersection, do so now. Otherwise press ENTER, and you’ll see the screen on the right, below.





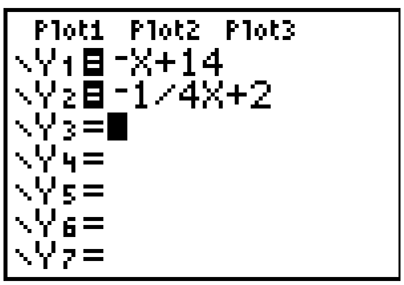
Because the point of intersection found on the calculator is (-6, 2), the solution of the system $\begin{cases} y = -\frac{2}{3}x - 2 \\ x - 2y = -10 \end{cases}$ is (-6, 2).

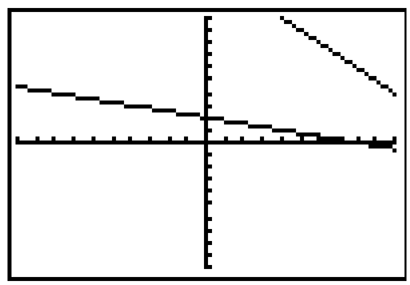
Example 2: Solve the system $\begin{cases} x + y = 14 \\ x + 4y = 8 \end{cases}$ graphically using the graphing calculator.

Solution: First, solve each equation for *y*.

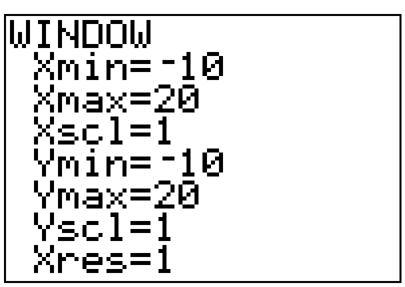
$$x + y = 14 \qquad\qquad x + 4y = 8$$
$$y = -x + 14 \qquad\qquad 4y = -x + 8$$
$$y = -\frac{1}{4}x + 2$$

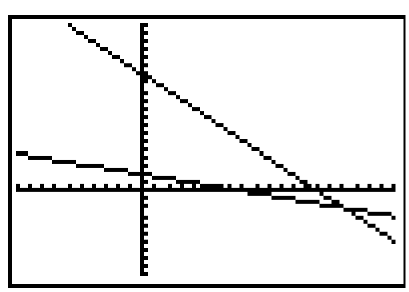
Next, graph $y = -x + 14$ and $y = -\frac{1}{4}x + 2$ in the same viewing rectangle on the calculator. Begin with the Standard (ZOOM 6) window. See the screens below.



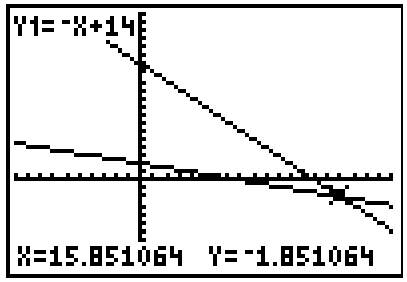


The point of intersection of the graphs, the solution of the system, lies off the screen. We need to change the viewing rectangle so that we can find the coordinates of the point of intersection of the two lines. The line $y = -x + 14$ has a *y*-intercept at 14, so change Ymax to 20. The solution is off the screen to the right too, so change Xmax to 20, and regraph the system. Advise students that they could also find the *x*-intercepts of both equations algebraically by setting *y* = 0 and solving for *x*.





Aha! We see the point of intersection of the graphs. Use the TRACE key to trace along one of the lines to determine the point of intersection.



Use the same steps as Example 1: put the cursor close to the point of intersection, and use 2ND TRACE to get to the CALC menu feature "intersect;" go through the three prompts, and find the point of intersection to be (16, -2), the solution of the system $\begin{cases} x + 2y = 4 \\ 2x - y = -7 \end{cases}$ is (16, -2).

INSTRUCTOR NOTE

Determining a suitable viewing rectangle can be a challenge for students. Encourage them to think about what they already know about an equation in two variables (*y*-intercept, for example), or what characteristics they can easily find (*x*-intercept). Many students need guidance with connecting what they already know about a graph with determining the viewing rectangle.

Example 3: Monique is creating a flyer to give to local businesses to advertise her new vintage clothing store. She is trying to decide between two quotes for the printing. *Short North Printers* has given her a quote of $70 setup fee plus $0.10 per flyer printed. *Beechwold Graphics* has given her a quote of $100 plus $0.04 per flyer.

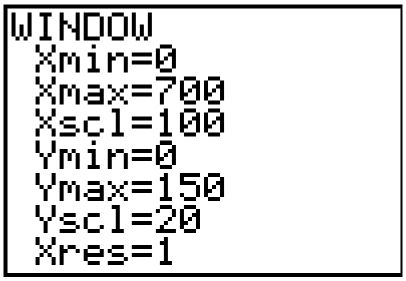
a. Write a system of linear equations that models the problem. Let x represent the number of flyers and y represent the total cost for printing x flyers.
b. Graph the system of equations to determine how many flyers she needs to have printed in order for the cost to be the same at each printer.
c. If 400 flyers are printed, which printer should she choose to have the lower cost?

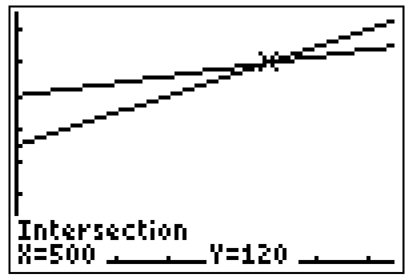
Solution: a. *Short North Printers*: $y = 70 + 0.10x$
Beechwold Graphics: $y = 100 + 0.04x$.

b. There's a lot to talk about when trying to determine a suitable viewing rectangle for this problem.

- What is a realistic number of flyers that Monique might print? Would she print a negative number of flyers?
- Will the total cost be a negative number?
- When the equations are expressed in slope-intercept form, what are the *y*-intercepts of each one?
- What does an ordered pair (x, y) represent in this problem?

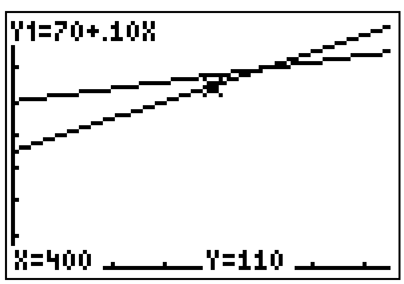
Here is one possible solution:

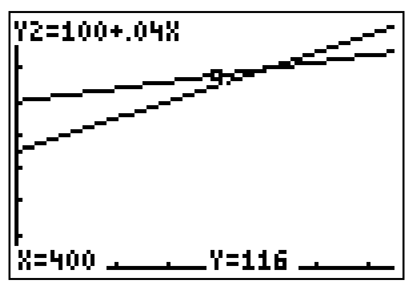




So when 500 flyers are printed, the cost for both *Short North Printers* and *Beechwold Graphics* was $120.

c. For 400 flyers, *Short North Printers* would charge $110, and *Beechwold Graphics* would charge $116. See the screen shots below. She should choose *Short North Printers*.





INSTRUCTOR NOTES

❖ Students can verify the solution of an inconsistent system, parallel lines, using the graphical representation. For example, $\begin{cases} 4x + \dfrac{y}{2} = 3 \\ y = -8x + 6 \end{cases}$.

❖ Students can verify the solution of a consistent system with dependent equations graphically. For example, $\begin{cases} y = \dfrac{1}{7}x + 3 \\ x - 7y = 21 \end{cases}$.