Math 2153 Homework #5 Key

1) a. $f(x,y) = x^3 - y^3$ $P(4,3)$ $\vec{v} = \frac{\sqrt{2}}{2}(\hat{\imath} + \hat{\jmath})$ (unit vector)

$\nabla f = 3x^2\,\hat{\imath} - 3y^2\,\hat{\jmath}$ $\nabla f(4,3) = 48\hat{\imath} - 27\hat{\jmath}$

$$\vec{\nabla} f \cdot \vec{v} = \langle 48, -27\rangle \cdot \left\langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right\rangle = 24\sqrt{2} - \frac{27}{2}\sqrt{2} = \sqrt{2}\left(24 - \frac{27}{2}\right) = \frac{21\sqrt{2}}{2}$$

b) $f(x,y) = \arccos(xy)$ $P(1,0)$ $\vec{v} = \hat{\imath} + 5\hat{\jmath}$ $\vec{u} = \frac{1}{\sqrt{26}}\hat{\imath} + \frac{5}{\sqrt{26}}\hat{\jmath}$

$\|\vec{v}\| = \sqrt{1+25} = \sqrt{26}$

$\nabla f = \frac{-y}{\sqrt{1-x^2y^2}}\hat{\imath} - \frac{x}{\sqrt{1-x^2y^2}}\hat{\jmath}$ $\nabla f(1,0) = -1\hat{\imath} + 0\hat{\jmath}$

$$\vec{\nabla} f \cdot \vec{u} = \langle -1, 0\rangle \cdot \left\langle \frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}} \right\rangle = \frac{-1}{\sqrt{26}}$$

c) $f(x,y,z) = xy + yz + xz$ $P(1,1,1)$ $\vec{v} = 2\hat{\imath} + \hat{\jmath} - \hat{k}$ $\vec{u} = \frac{2}{\sqrt{6}}\hat{\imath} + \frac{1}{\sqrt{6}}\hat{\jmath} - \frac{1}{\sqrt{6}}\hat{k}$

$\|\vec{v}\| = \sqrt{4+1+1} = \sqrt{6}$

$\vec{\nabla} f = (y+z)\hat{\imath} + (x+z)\hat{\jmath} + (x+y)\hat{k}$

$\vec{\nabla} f(1,1,1) = 2\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$

$$\vec{\nabla} f \cdot \vec{u} = \langle 2,2,2\rangle \cdot \left\langle \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}} \right\rangle = \frac{4}{\sqrt{6}} + \cancel{\frac{2}{\sqrt{6}}} - \cancel{\frac{2}{\sqrt{6}}} = \frac{4}{\sqrt{6}}$$

d) $f(x,y,z) = x\arctan(yz)$ $P(4,1,1)$ $\vec{v} = \hat{\imath} + 2\hat{\jmath} - \hat{k}$ $\vec{u} = \frac{1}{\sqrt{6}}\hat{\imath} + \frac{2}{\sqrt{6}}\hat{\jmath} - \frac{1}{\sqrt{6}}\hat{k}$

$\|\vec{v}\| = \sqrt{1+4+1} = \sqrt{6}$

$\vec{\nabla} f = \arctan(yz)\hat{\imath} + \frac{xz}{1+y^2z^2}\hat{\jmath} + \frac{xy}{1+y^2z^2}\hat{k}$

$\vec{\nabla} f(4,1,1) = \frac{\pi}{4}\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$

$$\vec{\nabla} f \cdot \vec{u} = \left\langle \frac{\pi}{4}, 2, 2 \right\rangle \cdot \left\langle \frac{1}{\sqrt{6}} + \frac{2}{\sqrt{6}} - \frac{1}{\sqrt{6}} \right\rangle = \frac{\pi}{4\sqrt{6}} + \frac{4}{\sqrt{6}} - \frac{2}{\sqrt{6}} = \frac{\pi}{4\sqrt{6}} + \frac{2}{\sqrt{6}} =$$

$$\frac{\pi}{4\sqrt{6}} + \frac{8}{4\sqrt{6}} = \frac{\pi + 8}{4\sqrt{6}}$$

e) $f(x,y) = \frac{y}{x+y}$ $\vec{v} = \cos(-\pi/6)\hat{\imath} + \sin(-\pi/6)\hat{\jmath} = \frac{\sqrt{3}}{2}\hat{\imath} - \frac{1}{2}\hat{\jmath}$

unit vector

$\vec{\nabla} f = \frac{-y}{(x+y)^2}\hat{\imath} + \frac{x + \cancel{y} - \cancel{y}}{(x+y)^2}\hat{\jmath}$ no point given

$$\vec{\nabla} f \cdot \vec{u} = \left\langle \frac{-y}{(x+y)^2}, \frac{x}{(x+y)^2} \right\rangle \cdot \left\langle \frac{\sqrt{3}}{2}, -\frac{1}{2} \right\rangle = \frac{-\sqrt{3}y}{2(x+y)^2} - \frac{x}{2(x+y)^2} = \frac{-\sqrt{3}y - x}{2(x+y)^2}$$

1) f. $f(x,y,z) = xye^z \quad P(2,4,0)\ ,\ Q(0,0,0)$

$\vec{v} = \langle -2, -4, 0 \rangle \quad \vec{u} = -\frac{1}{\sqrt{5}}\hat{\imath} - \frac{2}{\sqrt{5}}\hat{\jmath} + 0\hat{k}$

$\|\vec{v}\| = \sqrt{4+16+0} = 2\sqrt{5}$

$\nabla f =$

$ye^z\hat{\imath} + xe^z\hat{\jmath} + xye^z\hat{k}$

$\nabla f(2,4,0) = 4\hat{\imath} + 2\hat{\jmath} + 8\hat{k}$

$\vec{\nabla} f \cdot \vec{u} = \langle 4, 2, 8 \rangle \cdot \langle -\frac{1}{\sqrt{5}}, -\frac{2}{\sqrt{5}}, 0 \rangle = -\frac{4}{\sqrt{5}} - \frac{4}{\sqrt{5}} + 0 = -\frac{8}{\sqrt{5}}$

g) $f(x,y) = \sin 2x \cos y \quad P(0,0),\ Q(\frac{\pi}{2}, \pi)$

$\vec{v} = \frac{\pi}{2}\hat{\imath} + \pi\hat{\jmath}$

$\vec{u} = \frac{2}{\sqrt{5}\pi}\langle \frac{\pi}{2}, \pi \rangle = \frac{1}{\sqrt{5}}\hat{\imath} + \frac{2}{\sqrt{5}}\hat{\jmath}$

$\vec{\nabla} f = 2\cos 2x \cos y\,\hat{\imath} - \sin 2x \sin y\,\hat{\jmath} \quad \|\vec{v}\| = \sqrt{\frac{\pi^2}{4} + \pi^2} = \sqrt{\frac{5\pi^2}{4}} = \frac{\sqrt{5}\pi}{2}$

$\vec{\nabla} f(0,0) = 2\hat{\imath} - 0\hat{\jmath}$

$\vec{\nabla} f \cdot \vec{u} = \langle 2, 0 \rangle \langle \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}} \rangle = \frac{2}{\sqrt{5}}$

2a) $f(x,y) = 2xe^{y/x} \quad (2,1)$

$\nabla f = \left(2e^{y/x} + 2xe^{y/x}\left(-\frac{y}{x^2}\right)\right)\hat{\imath} + \left(2xe^{y/x} \cdot \frac{1}{x}\right)\hat{\jmath}$

$= 2e^{y/x}\left[\left(1 - \frac{y}{x}\right)\hat{\imath} + 1\hat{\jmath}\right]$

$\|\nabla f\| = 2e^{y/x}\sqrt{\left(1-\frac{y}{x}\right)^2 + 1}$ maximum directional derivative

$\|\nabla f(2,1)\| = 2e^{1/2}\sqrt{\left(1-\frac{1}{2}\right)^2 + 1} = 2e^{1/2}\sqrt{\frac{1}{4}+1} = 2e^{1/2}\sqrt{\frac{5}{4}} = 2e^{1/2}\frac{\sqrt{5}}{2} = e^{1/2}\sqrt{5}$

b) $f(x,y) = \ln(x^2 - y) \quad (2,3)$

$\nabla f = \frac{2x}{x^2-y}\hat{\imath} - \frac{1}{x^2-y}\hat{\jmath} \quad \nabla f(2,3) = \frac{4}{1}\hat{\imath} - \frac{1}{1}\hat{\jmath} = 4\hat{\imath} - 1\hat{\jmath}$

$\|\nabla f\| = \sqrt{16+1} = \sqrt{17}$

c) $f(x,y) = y\cos(x-y) \quad (0, \pi/3)$

$\nabla f = -y\sin(x-y)\hat{\imath} + \left(\cos(x-y) + y\sin(x-y)\right)\hat{\jmath}$

$\nabla f(0, \pi/3) = \frac{\pi}{3}\sin(-\pi/3)\hat{\imath} + \left(\cos(\pi/3) + \frac{\pi}{3}\sin(-\pi/3)\right)\hat{\jmath}$

$= \frac{\pi}{3}\left(-\frac{\sqrt{3}}{2}\right)\hat{\imath} + \left(\frac{1}{2} + \frac{\pi}{3}\left(-\frac{\sqrt{3}}{2}\right)\right)\hat{\jmath} = -\frac{\pi\sqrt{3}}{6}\hat{\imath} + \left(\frac{3-\pi\sqrt{3}}{6}\right)\hat{\jmath}$

$\|\nabla f\| = \sqrt{\frac{3\pi^2}{36} + \frac{9 - 6\pi\sqrt{3} + 3\pi^2}{36}} = \frac{\sqrt{6\pi^2 - 6\pi\sqrt{3} + 9}}{6}$

2 d) $f(x,y,z) = \frac{1}{\sqrt{1-x^2-y^2-z^2}} = (1-x^2-y^2-z^2)^{-1/2}$ $\quad (0,0,0)$

$\nabla f = -\frac{1}{2}(1-x^2-y^2-z^2)^{-3/2}(-2x)\hat{i} + \left(-\frac{1}{2}\right)(1-x^2-y^2-z^2)^{-3/2}(-2y)\hat{j} + \left(-\frac{1}{2}\right)(1-x^2-y^2-z^2)^{-3/2}(-2z)\hat{k}$

$= \frac{1}{(1-x^2-y^2-z^2)^{3/2}} \langle x, y, z \rangle \qquad \nabla f(0,0,0) = \vec{0}$

$\|\nabla f\| = 0$

e) $f(x,y,z) = xy^2z^2 \quad (2,1,1)$

$\nabla f = y^2z^2\,\hat{i} + 2xyz^2\,\hat{j} + 2xy^2z\,\hat{k}$

$\nabla f(2,1,1) = \hat{i} + 4\hat{j} + 4\hat{k} \qquad \|\nabla f\| = \sqrt{1+16+16} = \sqrt{33}$

3) to calculate the directional derivative, one need only calculate the gradient on an explicit function using only the independent variables

to calculate the normal vector, you need a vector pointing off of the surface and so all variables (including the dependent one) must be used. We do this by creating a function F by solving the original equation for 0, and taking the gradient of the function that results.

4) for this problem, we need the second method described above.

a) $z = x^2+y^2 \Rightarrow F(x,y,z) = z - x^2 - y^2 \qquad \nabla F = -2x\hat{i} - 2y\hat{j} + 1\hat{k}$

$z = 4-y \Rightarrow G(x,y,z) = z + y - 4 \qquad \nabla G = 0\hat{i} + 1\hat{j} + 1\hat{k}$

$\vec{\nabla} F(2,-1,5) = -4\hat{i} + 2\hat{j} + 1\hat{k} \qquad \vec{\nabla} G(2,-1,5) = 0\hat{i} + \hat{j} + 1\hat{k}$

$\vec{\nabla} F \cdot \vec{\nabla} G = \langle -4, 2, 1 \rangle \cdot \langle 0, 1, 1 \rangle = 0 + 2 + 1 = 3 \neq 0$ not orthogonal

(dot product is related to the cosine. if the vectors are orthogonal (i.e. $\perp$) their dot product is zero)

4 b) $x^2+z^2=25$ $\quad F(x,y,z)=x^2+z^2-25$ $\quad (3,3,4)$

$y^2+z^2=25$ $\quad G(x,y,z)=y^2+z^2-25$

$\nabla F = 2x\hat{\imath}+0\hat{\jmath}+2z\hat{k}$ $\quad \nabla F(3,3,4) = 6\hat{\imath}+8\hat{k}$

$\nabla G = 0\hat{\imath}+2y\hat{\jmath}+2z\hat{k}$ $\quad \nabla G(3,3,4) = 6\hat{\jmath}+8\hat{k}$

$\vec{\nabla}F\cdot\vec{\nabla}G = \langle 6,0,8\rangle\cdot\langle 0,6,8\rangle = 0+0+64 \neq 0$ not orthogonal

more generally, you can use this method to find points

a) $\langle -2x,-2y,1\rangle\cdot\langle 0,1,1\rangle = 0-2y+1=0$

$2y=1 \Rightarrow y=1/2$ at any point where $y=1/2$ the graphs will be orthogonal

likewise

b) $\langle 2x,0,2z\rangle\cdot\langle 0,2y,2z\rangle = 0+0+4z^2=0$

$z=0$ any point where $z=0$ they will be orthogonal.

5) $f(x,y,z,w)=3x^2+y^2+2z^2-5w^2$ s.t. $x+6y+3z+2w=4$

$F(x,y,z,w,\lambda)=3x^2+y^2+2z^2-5w^2-\lambda x-6\lambda y-3\lambda z-2\lambda w+4$

$F_x=6x-\lambda=0$ $\quad \lambda=6x$ $\quad x=\frac{\lambda}{6}$ $\quad x=\frac{\cancel{240}^{40}}{\cancel{6}(1201)}=\frac{40}{1201}$

$F_y=2y-6\lambda=0$ $\quad 2y=6\lambda$ $\quad y=3\lambda$ $\quad y=\frac{3(240)}{1201}=\frac{720}{1201}$

$F_z=4z-3\lambda=0$ $\quad 4z=3\lambda$ $\quad z=\frac{3}{4}\lambda$ $\quad z=\frac{3(\cancel{240}^{60})}{4(1201)}=\frac{180}{1201}$

$F_w=-10w-2\lambda=0$ $\quad -10w=2\lambda$ $\quad w=-\frac{1}{5}\lambda$ $\quad w=-\frac{\cancel{240}^{48}}{\cancel{5}(1201)}=\frac{-48}{1201}$

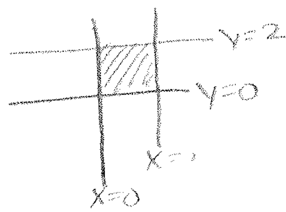
$\frac{\lambda}{6}+6(3\lambda)+3(\frac{3}{4}\lambda)+2(-\frac{1}{5}\lambda)=4$

$\frac{1}{6}\lambda+18\lambda+\frac{9}{4}\lambda-\frac{2}{5}\lambda=4 \Rightarrow$

$\frac{1201}{60}\lambda=4$ $\quad \lambda=\frac{4\cdot 60}{1201}=\frac{240}{1201}$

$(x,y,z,w)=\left(\frac{40}{1201},\frac{720}{1201},\frac{180}{1201},\frac{-48}{1201}\right)$ f optimum: $\frac{480}{1201}$

6) b) $\int_0^1\int_0^2 (x+y)\,dy\,dx$ region:

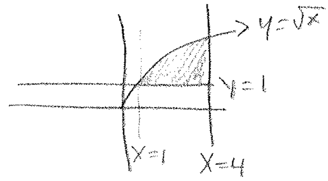


rectangle in the plane

$\int_0^1 xy+\frac{1}{2}y^2\Big|_0^2 dx = \int_0^1 2x+2\,dx$

$= x^2+2x\Big|_0^1 = 1+2 = 3$

6) i. $\int_1^4 \int_1^{\sqrt{x}} 2ye^{-x}\,dy\,dx$



$\int_1^4 y^2 e^{-x} \Big|_1^{\sqrt{x}} dx =$

$\int_1^4 xe^{-x} - e^{-x}\,dx = \int_1^4 (x-1)e^{-x}\,dx$

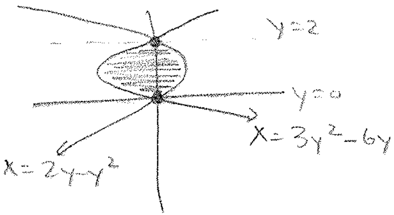
	u	dv
+	$x-1$	e^{-x}
−	1	$-e^{-x}$
+	0	e^{-x}

$-(x-1)e^{-x} - e^{-x} \Big|_1^4 = -3e^{-4} - e^{-4} + 0 + e^0$

$= -4e^{-4} + 1$

j) $\int_0^2 \int_{3y^2-6y}^{2y-y^2} 3y\,dx\,dy$

opens left
$x = 2y - y^2$
$0 = y(2-y)$
opens right
$x = 3y^2 - 6y$
$0 = 3y(y-2)$

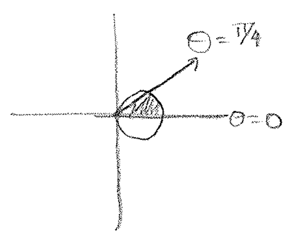


$\int_0^2 3xy \Big|_{3y^2-6y}^{2y-y^2} dy = \int_0^2 3(2y-y^2)y - 3(3y^2-6y)y\,dy =$

$\int_0^2 6y^2 - 3y^3 - 9y^3 + 18y^2\,dy = \int_0^2 24y^2 - 12y^3\,dy = 8y^3 - 3y^4 \Big|_0^2$

$= 64 - 48 = 12$

k) $\int_0^{\pi/4} \int_0^{\cos\theta} 3r^2 \sin\theta\,dr\,d\theta =$



$r=0$
$r=\cos\theta$
$\theta=0$

$\int_0^{\pi/4} r^3 \sin\theta \Big|_0^{\cos\theta} d\theta =$

$\int_0^{\pi/4} \cos^3\theta \sin\theta\,d\theta =$

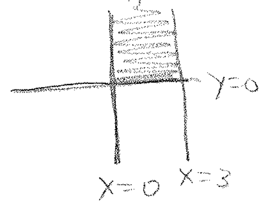
$u = \cos\theta$
$-du = \sin\theta\,d\theta$

$-\int u^3\,du = -\frac{1}{4}u^4 \Rightarrow -\frac{1}{4}\cos^4\theta \Big|_0^{\pi/4} = -\frac{1}{4}\left(\frac{1}{\sqrt{2}}\right)^4 + \frac{1}{4}(1) =$

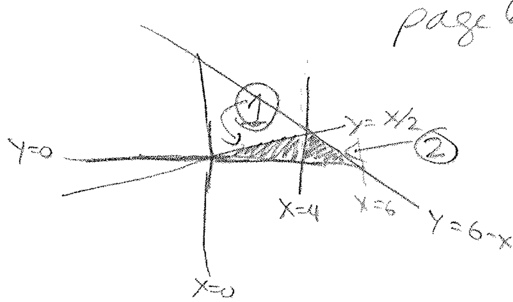
$-\frac{1}{4}\left(\frac{1}{4}\right) + \frac{1}{4} = \frac{1}{4} - \frac{1}{16} = \frac{3}{16}$

l) $\int_0^3 \int_0^\infty \frac{x^2}{1+y^2}\,dy\,dx = \int_0^3 x^2 \arctan y \Big|_0^\infty dx$

$= \int_0^3 \frac{\pi}{2} x^2\,dx = \frac{\pi}{2}\frac{1}{3}x^3 \Big|_0^3 = \frac{\pi}{2} \cdot 9 = \frac{9\pi}{2}$



6). m. $\int_0^4 \int_0^{x/2} dy\,dx + \int_4^6 \int_0^{6-x} dy\,dx$ ① ②

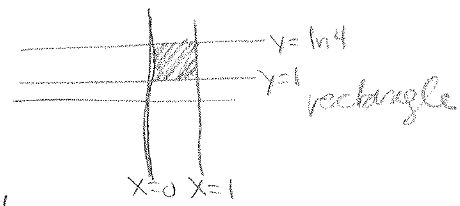


$\int_0^4 y\Big|_0^{x/2} dx + \int_4^6 y\Big|_0^{6-x} dx =$

$\int_0^4 \frac{x}{2}\,dx + \int_4^6 6-x\,dx = \frac{x^2}{4}\Big|_0^4 + \left(6x-\frac{1}{2}x^2\right)\Big|_4^6 =$

$4+36-18-24+8 = 6$

n. $\int_0^1\int_1^{\ln 4} \frac{\sinh(x)}{1+\sinh^2(y)}\,dy\,dx = \int_0^1\int_1^{\ln 4} \frac{\sinh(x)}{\cosh^2 y}\,dy\,dx$



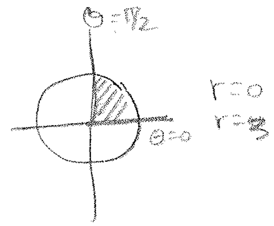
$= \int_0^1\int_1^{\ln 4} \sinh x \,\mathrm{sech}^2 y\,dy\,dx = \int_0^1 \sinh x \tanh y\Big|_1^{\ln 4} dx =$

$\int_0^1 \sinh x\,(\tanh(\ln 4)-\tanh(1))\,dy =$

$[\tanh(\ln 4)-\tanh(1)]\cosh x\Big|_0^1 = [\tanh(\ln 4)-\tanh(1)]\cdot(\cosh 1 - 1)$

$\approx .06558$

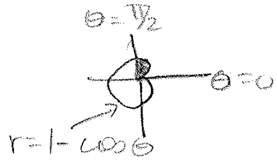
o. $\int_0^{\pi/2}\int_0^3 re^{-r^2}\,dr\,d\theta$



$\int_0^{\pi/2} -\frac{1}{2}e^{-r^2}\Big|_0^3 d\theta = -\frac{1}{2}(e^{-9}-e^0)\int_0^{\pi/2} d\theta$

$= \frac{1}{2}(1-e^{-9})\frac{\pi}{2} = \frac{\pi}{4}\left(1-\frac{1}{e^9}\right)$

P. $\int_0^{\pi/2}\int_0^{1-\cos\theta} \sin\theta\, r\,dr\,d\theta$



$\int_0^{\pi/2} \frac{1}{2}r^2\sin\theta\Big|_0^{1-\cos\theta} d\theta =$

$\frac{1}{2}\int_0^{\pi/2}(1-\cos\theta)^2\sin\theta\,d\theta \Rightarrow$ $u = 1-\cos\theta$, $du = \sin\theta\,d\theta$

$\frac{1}{2}\int u^2\,du = \frac{1}{6}u^3 \Rightarrow \frac{1}{6}(1-\cos\theta)^3\Big|_0^{\pi/2} = \frac{1}{6}[1^3-0^3] = \frac{1}{6}$

8) a. $\int_{-1}^{1}\int_{-1}^{1} (\sin x\cos y)\cdot(\cos x \sin y)\,dy\,dx = \int_{-1}^{1}\int_{-1}^{1}(\sin x\cos x)(\cos y\sin y)\,dy\,dx$

$\int_{-1}^{1} \sin x\cos x\,(\tfrac{1}{2}\sin^2 y)\Big|_{-1}^{1}dx = \tfrac{1}{2}\underbrace{(\sin^2(1)-\sin^2(-1))}_{=0}\int_{-1}^{1}\sin x\cos x\,dx = 0$ by symmetry.

these functions are orthogonal

b) $\int_{-1}^{1}\int_{-1}^{1} e^{y}\sin(2x)\cdot e^{y}\sin x\,dy\,dx = \int_{-1}^{1}\int_{-1}^{1} e^{2y}\sin 2x\sin x\,dy\,dx =$

$\int_{-1}^{1}\tfrac{1}{2}e^{2y}\Big|_{-1}^{1}\sin 2x\sin x\,dx = \tfrac{1}{2}(e^{2}-e^{-2})\int_{-1}^{1}\sin 2x\sin x\,dx =$

$\tfrac{1}{2}(e^{2}-\tfrac{1}{e^{2}})\int_{-1}^{1}\tfrac{1}{2}(\cos x-\cos 3x)\,dx = \tfrac{1}{4}(e^{2}-\tfrac{1}{e^{2}})\int_{-1}^{1}\cos x-\cos 3x\,dx =$

$\tfrac{1}{4}(e^{2}-\tfrac{1}{e^{2}})[\sin x-\tfrac{1}{3}\sin 3x]_{-1}^{1} = \tfrac{1}{4}(e^{2}-\tfrac{1}{e^{2}})[\sin(1)-\sin(-1)-\tfrac{1}{3}\sin(3)+\tfrac{1}{3}\sin(-3)]$

$\tfrac{1}{4}(e^{2}-\tfrac{1}{e^{2}})(\sin(1)+\underbrace{\sin(1)-\tfrac{1}{3}\sin(3)}_{\text{by symmetry}}-\tfrac{1}{3}\sin(3)] = \tfrac{1}{4}(e^{2}-\tfrac{1}{e^{2}})(2\sin(1)-\tfrac{2}{3}\sin(3))$

$\neq 0$

no orthogonal

c) $\int_{-1}^{1}\int_{-1}^{1}\sinh x\cosh y\,dy\,dx = \int_{-1}^{1}\sinh x\sinh y\Big|_{-1}^{1}dx =$

$\int_{-1}^{1}\sinh x\,(\sinh(1)-\sinh(-1))\,dx = \int_{-1}^{1}\sinh(x)\,(\sinh(1)+\sinh(1))\,dx$

by symmetry

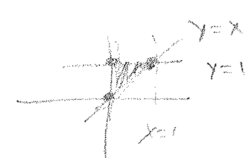
$= 2\sinh(1)\int_{-1}^{1}\sinh x\,dx = 2\sinh(1)\cosh x\Big|_{-1}^{1} =$

$2\sinh(1)[\cos(1)-\cosh(-1)] = 2\sinh(1)[\cosh 1-\cosh(1)] = 0$

by symmetry

These are orthogonal

8) a. $2\int_{0}^{1}\int_{x}^{1}\sin(x+y)\,dy\,dx =$

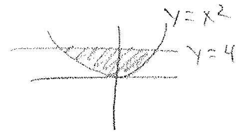


$-2\int_{0}^{1}\cos(x+y)\Big|_{x}^{1}dx = -2\int_{0}^{1}\cos(x+1)-\cos(2x)\,dx$

$-2[\sin(x+1)-\tfrac{1}{2}\sin(2x)]\Big|_{0}^{1} = -2[\sin(2)-\tfrac{1}{2}\sin 2-\sin(1)+0]$ X=0 $A=\tfrac{1}{2}(1)(1)=\tfrac{1}{2}$

$= -2[\tfrac{1}{2}\sin(2)-\sin(1)] = 2\sin(1)-\sin(2)$

8) b. $A = 2\int_0^2 \int_{x^2}^4 dy\,dx = 2\int_0^2 4-x^2\,dx =$



$2\left[4x - \frac{1}{3}x^3\right|_0^2 = 2\left[8 - \frac{8}{3}\right] = \frac{32}{3}$

$\bar{f} = \frac{3}{32}\int_{-2}^2 \int_{x^2}^4 \sin^2 x\,dy\,dx = \frac{3}{32}\int_{-2}^2 (4-x^2)(\sin^2 x)\,dx =$

$\frac{3}{32}\int_{-2}^2 (4-x^2)\left(\frac{1}{2}\right)(1-\cos 2x)\,dx = \frac{3}{64}\int_{-2}^2 (4-x^2)(1-\cos 2x)\,dx$

by symmetry $\frac{3}{32}\int_0^2 (4-x^2)(1-\cos 2x)\,dx = \frac{3}{32}\int_0^2 4 - x^2 - 4\cos 2x + 4x^2\cos 2x\,dx$

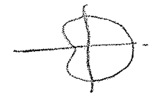
$\frac{3}{32}\left[4x - \frac{1}{3}x^3 - 2\sin 2x + 2x^2\sin 2x + 2x\cos 2x - \sin 2x\right|_0^2$

	u	dv
+	$4x^2$	$\cos 2x$
−	$8x$	$\frac{1}{2}\sin 2x$
+	8	$-\frac{1}{4}\cos 2x$
	0	$-\frac{1}{8}\sin 2x$

$\frac{3}{32}\left[8 - \frac{8}{3} - 2\sin(4) + 8\sin(4) + 4\cos(4) - \sin(4)\right] =$

$\frac{3}{32}\left[\frac{16}{3} + 5\sin(4) + 4\cos 4\right] = \frac{1}{2} + \frac{15}{32}\sin(4) + \frac{3}{8}\cos(4)$

c. $\frac{1}{A}\int_0^{2\pi}\int_0^{2+\cos\theta} \cosh(r^2)\,r\,dr\,d\theta =$ $\quad u = r^2 \quad du = 2r\,dr \quad \frac{1}{2}du = r\,dr$



$\frac{1}{2}\int \cosh u\,du = \frac{1}{2}\sinh(u) \Rightarrow$

$\frac{\cancel{2}}{9\pi}\int_0^{2\pi} \frac{1}{\cancel{2}}\sinh(2+\cos\theta)\,d\theta =$

$\frac{1}{9\pi}\int_0^{2\pi} \sinh(2+\cos\theta)\,d\theta$

this can only be done numerically

≈ 1.0204

$A = \int_0^{2\pi}\int_0^{2+\cos\theta} r\,dr\,d\theta$

$\int_0^{2\pi} \frac{1}{2}r^2\Big|_0^{2+\cos\theta} d\theta =$

$\frac{1}{2}\int_0^{2\pi} 4 + 4\cos\theta + \cos^2\theta\,d\theta$

$= \frac{1}{2}\int_0^{2\pi} 4 + 4\cos\theta + \frac{1}{2} + \frac{1}{2}\cos 2\theta\,d\theta$

$= \frac{1}{2}\left[\frac{9}{2}\theta + \cancel{4\sin\theta}^{0} + \frac{1}{4}\cancel{\sin 2\theta}^{0}\right]_0^{2\pi} =$

$\frac{9}{4}[2\pi] = \frac{9\pi}{2}$