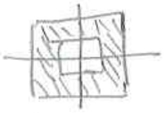
Math 2153, Quiz #15, Fall 2012 Name KEY

Instructions: Show all work. Use exact answers unless specifically asked to round.

1. Explain why the line integral $\int_C 3x^2e^y dx + e^y dy$ on the path $C: boundary\ of\ region\ lying\ between\ the\ squares\ with\ vertices\ (1,1), (-1,1), (-1,-1), (1,-1) and\ (2,2), (-2,2), (-2,-2), (2,-2)$ should be done with Green's Theorem? Then find the value of the integral by that method.

$$\frac{\partial N}{\partial x} = 0 \qquad \frac{\partial M}{\partial y} = 3x^2e^y \qquad \int_{-2}^{2}\int_{-2}^{2} -3x^2e^y\,dy\,dx - \int_{-1}^{1}\int_{-1}^{1} -3x^2e^y\,dy\,dx$$



$$\int_{-2}^{2} -3x^2e^y\Big|_{-2}^{2} dx + \int_{-1}^{1} 3x^2e^y\Big|_{-1}^{1} dx =$$

$$\int_{-2}^{2} -3x^2(e^2-e^{-2})dx + \int_{-1}^{1} 3x^2(e-1/e)\,dx$$

$$-x^3\Big|_{-2}^{2}(e^2-1/e^2) + x^3(e-1/e)\Big|_{-1}^{1} =$$

$$-(8-(-8))(e^2-1/e^2) + (1-(-1))(e-1/e) =$$

$$\boxed{-16(e^2-1/e^2)+2(e-1/e)}$$

2. Consider the vector valued function $\vec{r}(u,v) = 2u\cosh(v)\vec{\imath} + 2u\sinh(v)\vec{\jmath} + \frac{1}{2}u^2\vec{k}$. Find the normal vector to the surface and state the equation of the tangent plane at the point (-4,0,2).

$$\vec{r}_u = 2\cosh v\,\hat{\imath} + 2\sinh v\,\hat{\jmath} + u\hat{k}$$

$$\vec{r}_v = 2u\sinh v\,\hat{\imath} + 2u\cosh v\,\hat{\jmath} + 0\hat{k}$$

$$\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2\cosh v & 2\sinh v & u \\ 2u\sinh v & 2u\cosh v & 0 \end{vmatrix} = (0-2u^2\cosh v)\hat{\imath} - (0-2u^2\sinh v)\hat{\jmath} + (4u\cosh^2 v - 4u\sinh^2 v)\hat{k}$$

$$-2u^2\cosh v\,\hat{\imath} + 2u^2\sinh v\,\hat{\jmath} + 4u\hat{k}$$

Since $\cosh^2 v - \sinh^2 v = 1$

$2u\cosh v = -4$

$2u\sinh v = 0 \Rightarrow \sinh v = 0$

$v = 0$

$\frac{1}{2}u^2 = 2$

$u^2 = 4$

$u = \pm 2$

Check

$\cosh(0) = 1$

$2u = -4$

$u = -2$

$(-2,0) \rightarrow$

$-8\hat{\imath} + 0\hat{\jmath} - 8\hat{k}$

tangent plane

$$\boxed{-8(x+4) + 0(y-0) - 8(z-2) = 0}$$