

Instructions: Show all work. Use exact values unless specifically told to round.

1. Find the critical points for the function $f(x, y) = x^3 + 6xy + 10y^2 + 4y + 4$, and characterize each point as either a maximum, minimum, saddle point, or cannot be determined.

$$f_x = 3x^2 + 6y = 0 \quad x^2 = -2y \quad -\frac{1}{2}x^2 = y$$

$$f_y = 6x + 20y + 4 = 0$$

$$\underline{6x - 10x^2 + 4 = 0}$$

$$5x^2 - 3x - 2 = 0$$

$$(5x + 2)(x - 1) = 0$$

$$x = -\frac{2}{5} \quad x = 1$$

$$x = 1 \quad -\frac{1}{2}(1)^2 = -\frac{1}{2} \quad (1, -\frac{1}{2})$$

$$x = -\frac{2}{5} \quad -\frac{1}{2}\left(-\frac{2}{5}\right)^2 = -\frac{1}{2} \cdot \frac{4}{25} = -\frac{2}{25} \quad \left(-\frac{2}{5}, -\frac{2}{25}\right)$$

$$f_{xx} = 6x$$

$$f_{yy} = 20$$

$$f_{xy} = 6$$

$$D(x, y) = f_{xx} \cdot f_{yy} - (f_{xy})^2$$

$$D(1, -\frac{1}{2}) = 6(20) - 6^2 > 0 \quad f_{xx} > 0 \cup \text{MIN}$$

$$D\left(-\frac{2}{5}, -\frac{2}{25}\right) = \left(-\frac{12}{5}\right)(20) - 6^2 < 0 \quad \text{Saddle point}$$