Math B1131, Quiz #4, Fall 2012 Name KEY

Instructions: Show all work. Give exact values unless specifically asked to round.

1. Use the limit process to calculate $\int_1^5 2x + 5\,dx$. Then verify your answer by the Fundamental Theorem of Calculus.

$$\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

① $\Delta x = \frac{b-a}{n} = \frac{5-1}{n} = \frac{4}{n}$

② $x_i = a + i\Delta x = 1 + \frac{4i}{n}$

③ $f(x_i) = 2\left(1+\frac{4i}{n}\right)+5 = 2 + \frac{8i}{n} + 5 = 7 + \frac{8i}{n}$

④ $\sum_{i=1}^{n} f(x_i)\Delta x = \sum_{i=1}^{n}\left(7+\frac{8i}{n}\right)\frac{4}{n} = \sum_{i=1}^{n}\left(\frac{28}{n}+\frac{32i}{n^2}\right) = \sum_{i=1}^{n}\frac{28}{n} + \sum_{i=1}^{n}\frac{32i}{n^2}$

⑤ $\sum_{i=1}^{n}\frac{28}{n} + \frac{32}{n^2}\sum i = \frac{28}{n}\cdot n + \frac{32}{n^2}\left(\frac{n(n+1)}{2}\right) = 28 + \frac{32n^2+32n}{2n^2}$

⑥ $\lim_{n\to\infty} 28 + \frac{\cancel{32}^{16}\cancel{n^2}}{\cancel{2n^2}} + \cancel{\frac{32n}{2n^2}}^{\to 0} = 28 + 16 = \boxed{44}$

FTC:

$$\int_1^5 2x+5\,dx = x^2+5x\Big|_1^5 = (25+25)-(1+5) = 50-6 = \boxed{44}$$