

Math 1020, Exam #1, Fall 2013

Name KEY

Instructions: Show all work. Use exact answers unless specifically asked to round. Problems with no work cannot receive partial credit.

1. For the numbers in the set $\left\{\frac{55}{7}, -11, 4, \sqrt{64}, -6.75, 14000, \sqrt{19}, \pi^2, \frac{0}{3}, 0.\overline{69}, \frac{16}{8}\right\}$, determine which of the numbers belongs to each of the number types below. (20 points)

a. Natural Numbers

$$\left\{4, \sqrt{64}, 14000, \frac{16}{8}\right\}$$

b. Integers

$$\left\{-11, 4, \sqrt{64}, 14000, \frac{0}{3}, \frac{16}{8}\right\}$$

c. Rational Numbers

$$\left\{\frac{55}{7}, -11, 4, \sqrt{64}, -6.75, 14000, \frac{0}{3}, 0.\overline{69}, \frac{16}{8}\right\}$$

d. Irrational Numbers

$$\left\{\sqrt{19}, \pi^2\right\}$$

2. $(3 + 4) + 5 = 3 + (4 + 5)$ is an example of which property of real numbers? (4 points)

associative property (of addition)

3. Simplify the expression. (6 points each)

a. $3(4q + 1) - 2(5q - 9)$

$$12q + 3 - 10q - 18$$

$$2q - 15$$

b. $\frac{1}{2}(4k - 6) + \frac{2}{3}(9 - 12k)$

$$2k - 3 + 6 - 8k$$

$$-6k + 3$$

4. Evaluate the expression $\frac{4xy-2z^2}{3xz+y^3}$ for $x = 1, y = -2, z = 3$. (7 points)

$$\frac{4(1)(-2) - 2(3)^2}{3(1)(3) + (-2)^3} = \frac{-8 - 2(9)}{9 + (-8)} = \frac{-8 - 18}{1} = -26$$

5. Explain why division into zero (ex. $\frac{0}{3}$) is equal to zero, but division by zero (ex. $\frac{3}{0}$) is not defined. (6 points)

if $x = \frac{0}{3}$ then this is related to the multiplication equation $3x = 0$. what times 3 is equal to 0? just 0.

but if $x = \frac{3}{0}$ this is related to the multiplication problem $0 \cdot x = 3$ or $0 = 3$ which is a contradiction. nothing times zero will ever be 3.

6. Simplify the following expression as much as possible. $\left(\frac{4^2 - 2(-13)(1)}{7 \cdot 3}\right)^2$ (7 points)

$$\left(\frac{16 + 26}{21}\right)^2 = \left(\frac{42}{21}\right)^2 = (2)^2 = 4$$

7. The following problem below has at least one error in it (possibly more than one). Find the error(s), explain why they are wrong, and solve the problem correctly. (15 points)

$$\frac{1}{4}(8x - 12) - \frac{1}{6}(18x - 24)$$

① Multiplication not allowed since this isn't an equation

$$12 \left[\frac{1}{4}(8x - 12) - \frac{1}{6}(18x - 24) \right]$$

$$3(8x - 12) - 2(18x - 24)$$

$$24x - 36 - 36x - 48$$

$$-12x - 84$$

$$-12x = 84$$

$$\frac{-12}{-12} \quad \frac{-12}{-12}$$

$$x = -7$$

② distributing the negative would result in +48

③ inserting an equal sign where there wasn't one before isn't allowed.

$$\frac{1}{4}(8x - 12) - \frac{1}{6}(18x - 24)$$

$$2x - 3 - (3x - 4)$$

$$2x - 3 - 3x + 4$$

$$-x + 1$$

8. What is the difference between "simplifying an expression" and "solving an equation"? Are there some things you can do in one of these that you cannot do in the other? (10 points)

in simplifying you can use properties of real numbers to reduce ~~the~~ the complexity of an equation, but one cannot obtain expressions for the variables.

Solving an equation allows additional operations like adding to both sides of the equation, or multiplying both sides of an equation, and determining specific values for any variables.

9. Solve the equations. (8 points each)

a. $\left(\frac{19}{9}x + \frac{2}{15} = -\frac{1}{3}x - \frac{19}{45}\right) 45$

~~$\frac{5}{45} \cdot \frac{19}{9}x + \frac{2}{15} \cdot \frac{45}{45} = -\frac{1}{3} \cdot \frac{15}{45}x - \frac{19}{45} \cdot \frac{45}{45}$~~

$5 \cdot 19x + 3 \cdot 2 = 15 \cdot (-x) - 19$

$95x + 6 = -15x - 19$
 $\quad \quad -6 \quad \quad \quad -6$

$95x = -15x - 25$
 $+15x \quad \quad +15x$

$110x = -25$

$\frac{110x}{110} = \frac{-25}{110} = -\frac{5}{22}$

$x = -\frac{5}{22}$

b. $\left(\frac{2x-9}{7} - 1 = \frac{4}{3}x\right) 21$

~~$\frac{3}{1} \cdot \left(\frac{2x-9}{7}\right) - 21 \cdot 1 = \frac{7}{1} \cdot \frac{4}{3}x$~~

$3(2x-9) - 21 = 7 \cdot 4x$

$6x - 9 - 21 = 28x$

$6x - 30 = 28x$
 $-6x \quad \quad -6x$

$-\frac{30}{22} = \frac{22x}{22}$

$-\frac{30}{22} = x$

$x = -\frac{15}{11}$

10. Below are three solved linear equations. For each one state the solution set, and whether the equation is an identity, a conditional equation, or a contradiction. (6 points each)

$3(x+4) = 2x + 7(x-1)$

$3x + 12 = 2x + 7x - 7$

$3x + 12 = 9x - 7$

a. $-6x + 12 = -7$

$-6x = -19$

$x = \frac{19}{6}$

Conditional equation

$x = \left\{ \frac{19}{6} \right\}$

$$6(x-2)+11=4(x-1)+2x$$

b. $6x-12+11=4x-4+2x$

$$6x-1=6x-4$$

$$-1=-4$$

Contradiction

x has no solutions
 \emptyset

$$\frac{1}{2}(2x+22)-1=3(x+9)-(2x+17)$$

c. $x+11-1=3x+27-2x-17$

$$x+10=x+10$$

identity

x is all real numbers

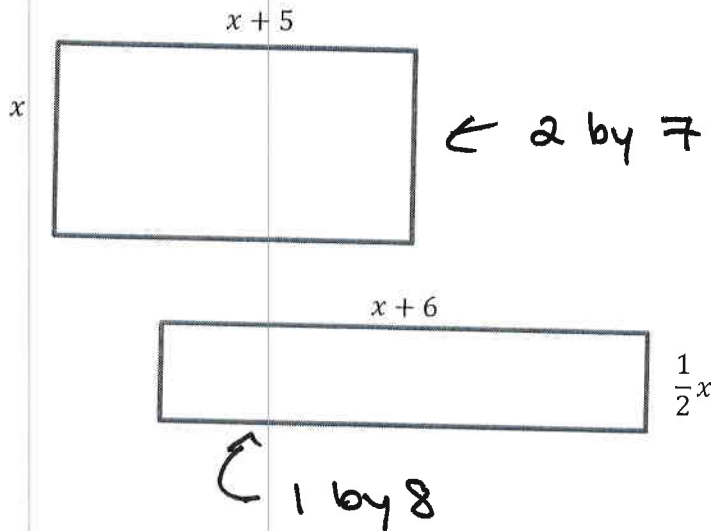
11. The two rectangles in the picture have the same perimeter. Solve the equation $2x + 2(x + 5) = 2(\frac{1}{2}x) + 2(x + 6)$, then find the dimensions of both rectangles. (12 points)

$$2x+2x+10 = x+2x+12$$

$$\begin{array}{r} 4x+10 = 3x+12 \\ -10 \quad -10 \end{array}$$

$$\begin{array}{r} 4x = 3x+2 \\ -3x \quad -3x \end{array}$$

$$x = 2$$



12. (Bonus question) Simplify the expression. (8 points)

$$\left(2\frac{1}{3} - 1\frac{5}{6}\right) + \frac{7}{8} \cdot \frac{4}{5}$$

$$\left(\frac{7}{3} - \frac{11}{6}\right) + \frac{7}{\cancel{8}^2} \cdot \frac{\cancel{4}^1}{5}$$

$$\left(\frac{2}{2} \cdot \frac{7}{3} - \frac{11}{6}\right) + \frac{7}{10}$$

$$\left(\frac{14}{6} - \frac{11}{6}\right) + \frac{7}{10}$$

$$\left(\frac{3}{6}\right) + \frac{7}{10}$$

$$\frac{1}{2} + \frac{7}{10}$$

$$\frac{1}{2} \cdot \frac{5}{5} + \frac{7}{10}$$

$$\frac{5}{10} + \frac{7}{10}$$

$$\frac{12}{10}$$

$$\boxed{\frac{6}{5}}$$