

# Conic Sections Key

## Circles

i.  $(x-3)^2 + (y-8)^2 = 1$

j.  $(x+5)^2 + (y+4)^2 = 9/4$

k.  $(x-1)^2 + (y+2)^2 = 16 \Rightarrow x^2 - 2x + 1 + y^2 + 4y + 4 - 16 = 0$   
 $\Rightarrow x^2 + y^2 - 2x + 4y - 11 = 0$

l.  $(x+4)^2 + (y-1)^2 = 178$

$r = \sqrt{(-4-9)^2 + (-1-2)^2} = \sqrt{169 + 9} = \sqrt{178}$

m.  $d = \sqrt{(-3-5)^2 + (4-2)^2} = \sqrt{64 + 36} = \sqrt{100} = 10$   $r = 5$

midpoint  $(\frac{-3+5}{2}, \frac{4-2}{2}) = (1, 1)$

$(x-1)^2 + (y-1)^2 = 25$

n.  $(x^2 + 4x + 4) + (y^2 + 6y + 9) = 18 + 4 + 9$

$(x+2)^2 + (y+3)^2 = 31$

Center  $(-2, -3)$   $r = \sqrt{31}$

o.  $x^2 + y^2 + 2x - 3/2y - 7/2 = 0$

$(x^2 + 2x + 1) + (y^2 - 3/2y + 9/16) = 7/2 + 1 + 9/16 = 81/16$

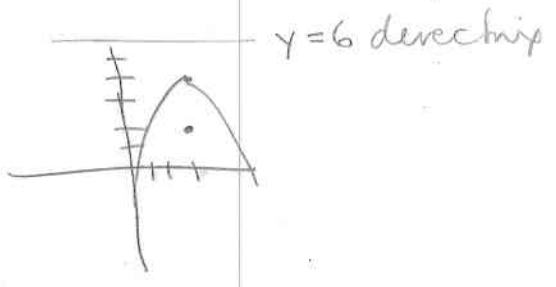
$(x+1)^2 + (y-3/4)^2 = 81/16$  Center  $(-1, 3/4)$  radius =  $9/4$

## parabola

i.  $a = 4 - 2 = 2$

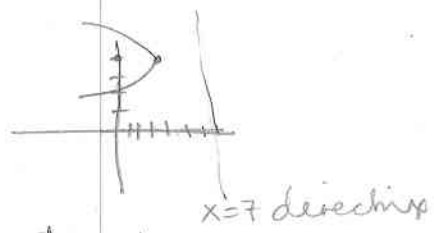
$(x-k)^2 = -4a(y-h)$

$(x-3)^2 = -8(y-4)$

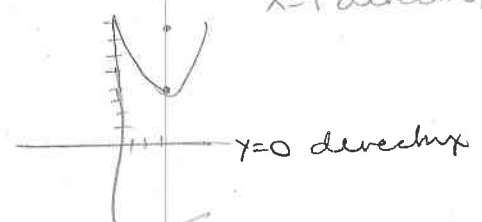


parabola continued

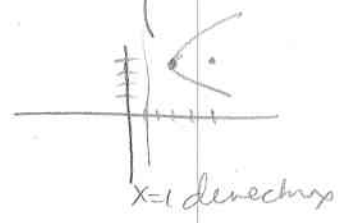
m.  $(y-k)^2 = -4a(x-h)$   $a=3$   
 $(y-4)^2 = -12(x-3)$



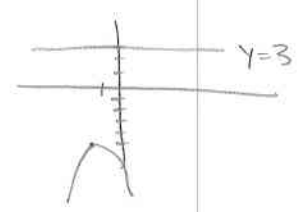
n.  $(x-h)^2 = 4a(y-k)$   $a=4$   
 $(x-3)^2 = 16(y-4)$



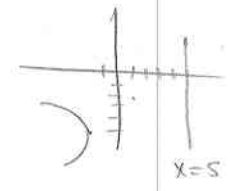
o.  $(y-k)^2 = 4a(x-h)$   $a=2$   
 $(y-4)^2 = 8(x-3)$



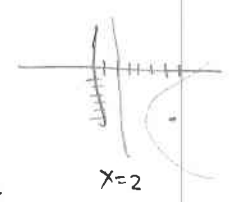
p.  $a=8$   
 $(x-h)^2 = -4a(y-k)$   
 $(x+1)^2 = -32(y+5)$



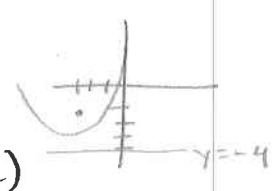
q.  $a=6$   
 $(y-k)^2 = -4a(x-h)$   
 $(y+5)^2 = -24(x+1)$



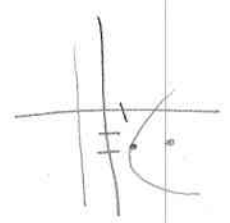
r.  $(y-k)^2 = 4a(x-h)$   $2a=5$   
 $(y+b)^2 = 10(x-\frac{9}{2})$   $a=\frac{5}{2}$   
 $2+\frac{5}{2} = \frac{4}{2} + \frac{5}{2} = \frac{9}{2}$   
 vertex at  $(\frac{9}{2}, -6)$



s.  $(x-h)^2 = 4a(y-k)$   $2a=3$   
 $(x+3)^2 = 6(y+\frac{3}{2})$   $a=\frac{3}{2}$   
 $-1-\frac{3}{2} = -\frac{5}{2}$   
 vertex  $(-3, -\frac{5}{2})$



t.  $y^2 + 4y + 4 = 8x - 12 + 4 = 8x - 8$   
 $(y+2)^2 = 8(x-1)$   $a=2$   
 vertex =  $(1, -2)$   
 focus  $(3, -2)$   
 directrix  $x=-1$



parabolas cont'd.

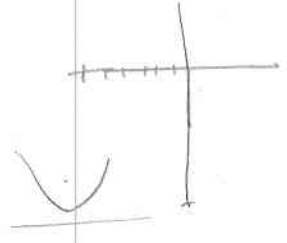
u.  $2x^2 + 24x - 6y - 20 = 0$

$x^2 + 12x + 36 = 3y + 10 + 36 = 3y + 46$

$(x+6)^2 = 3(y + \frac{46}{3})$        $a = \frac{3}{4}$  vertex  $(-6, -\frac{46}{3})$

focus  $(-6, -\frac{175}{12})$

directrix  $y = -\frac{95}{4}$



ellipse

n.  $\frac{(x-2)^2}{9} + \frac{(y+1)^2}{25} = 1$

$a=3$        $b=5$

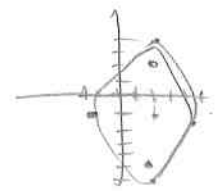
Center  $(2, -1)$

minor axis  $(-1, -1)$   $(5, -1)$

major axis  $(2, 4)$   $(2, -6)$

foci  $(2, 3)$   $(2, -5)$

$a^2 - b^2 = c^2$   
 $25 - 9 = 16 \Rightarrow c = 4$



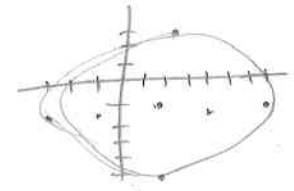
o.  $\frac{(x-2)^2}{25} + \frac{(y+1)^2}{16} = 1$

$a=5$       Center  $(2, -1)$   
 $b=4$   
 $c=3$

major axis  $(-3, -1)$ ,  $(7, -1)$

minor axis  $(2, 3)$   $(2, -5)$

foci  $(-1, -1)$ ,  $(5, -1)$



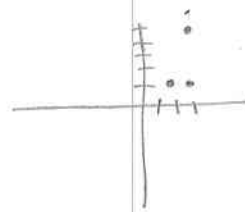
p.  $\frac{(x-3)^2}{1} + \frac{(y-1)^2}{10} = 1$

$c=3$   
 $b=1$   
 $1^2 + 3^2 = 10$   
 $a = \sqrt{10}$

major axis  $(3, 1 + \sqrt{10})$

minor axis  $(2, 1)$ ,  $(4, 1)$

foci  $(3, 4)$ ,  $(3, -2)$



ellipses, cont'd

$$g. \frac{(x+4)^2}{15} + \frac{(y-5)^2}{16} = 1$$

$$c=1$$

$$a=4$$

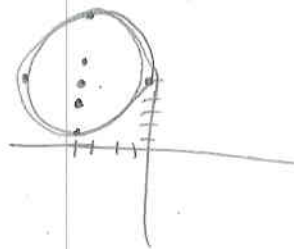
$$16-1=15$$

$$b=\sqrt{15}$$

major axis  $(-4, 1)$   $(4, 9)$

minor axis  $(-4 \pm \sqrt{15}, 5)$

foci  $(-4, 4)$   $(-4, 6)$



$$r. \frac{(x+3)^2}{9} + \frac{(y+2)^2}{36} = 1$$

$$a=6$$

$$b=3$$

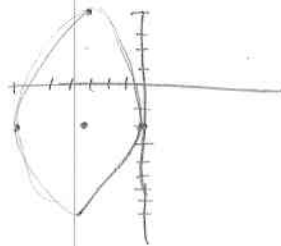
$$36-9=27$$

$$c=3\sqrt{3}$$

major axis  $(-3, 4)$ ,  $(-3, -8)$

minor axis  $(0, -2)$ ,  $(-6, -2)$

foci  $(-3, -2 \pm 3\sqrt{3})$



$$s. a=4 \quad c=3 \quad 16-9=7 \quad b=\sqrt{7}$$

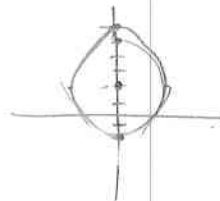
center  $(0, 2)$

$$\frac{x^2}{7} + \frac{(y-2)^2}{16} = 1$$

major axis  $(0, 6)$ ,  $(0, -2)$

minor axis  $(\pm\sqrt{7}, 2)$

foci  $(0, 5)$   $(0, -1)$



$$t. \frac{3x^2}{3} + \frac{y^2 - 2y + 1}{3} = \frac{2+1}{3}$$

$$\frac{x^2}{1} + \frac{(y-1)^2}{3} = 1$$

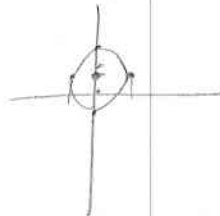
center  $(0, 1)$

$$a=\sqrt{3}, \quad b=1 \quad c=\sqrt{2}$$

major axis  $(0, 1 \pm \sqrt{3})$

minor axis  $(\pm 1, 1)$

foci  $(0, 1 \pm \sqrt{2})$



ellipses, cont'd.

u.  $21x^2 + 8y^2 + 84x - 48y + 44 = 0$

$21(x^2 + 4x + 4) + 8(y^2 - 6y + 9) = -44 + 84 + 72 = 112$

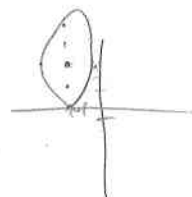
$\frac{(x+2)^2}{\frac{16}{3}} + \frac{(y-3)^2}{14} = 1 \Rightarrow$

Center  $(-2, 3)$   $a = \sqrt{14}$   $b = \frac{4}{\sqrt{3}}$   
 $14 - \frac{16}{3} = \frac{26}{3} = c$

major axis  $(-2, 3 \pm \sqrt{14})$

minor axis  $(-2 \pm \frac{4}{\sqrt{3}}, 3)$

foci  $(-2, 3 \pm \sqrt{\frac{26}{3}})$



v.  $225x^2 + 144y^2 + 450x - 576y + 701 = 0$

$225(x^2 + 2x + 1) + 144(y^2 - 4y + 4) = -701 + 225 + 576 = 100$

$\frac{(x+1)^2}{\frac{4}{9}} + \frac{(y-2)^2}{\frac{25}{36}} = 1$

$a = \frac{5}{6}$   $b = \frac{2}{3}$

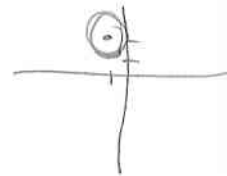
$\frac{25}{36} - \frac{4}{9} = \frac{1}{4}$   $c = \frac{1}{2}$

Center  $(-1, 2)$

major axis  $(-1, \frac{17}{6})$   $(-1, \frac{17}{6})$

minor axis  $(-\frac{5}{3}, 2)$   $(-\frac{1}{3}, 2)$

foci  $(-1, \frac{5}{2})$   $(-1, \frac{3}{2})$



w. domain  $[-3, 7]$

x. y-symmetric.

hyperbola

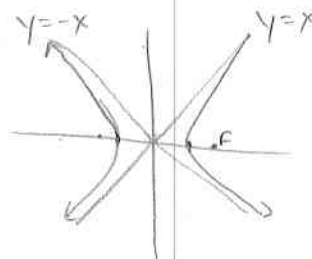
d.  $x^2 - y^2 = 1$

Center  $(0, 0)$

$a = 1$   $b = 1$

$c = \sqrt{2}$

foci  $(\pm\sqrt{2}, 0)$  vertices  $(\pm 1, 0)$



transverse - x

Hyperbolas, cont'd

M.  $\frac{(y-6)^2}{1} - \frac{(x+1)^2}{4} = 1$

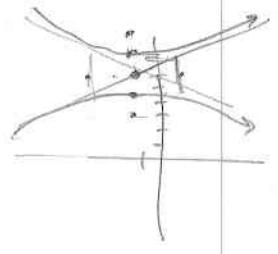
a=1 b=2 c=√5

Center (-1, 6) transverse y

vertices (-1, 7) (-1, 5)

foci (-1, 6 ± √5)

asymptotes y-6 = ± 1/2 (x+1)



N.  $\frac{(x+3)^2}{5} - \frac{(y-1)^2}{7} = 1$

a=√5, b=√7

5+7=12

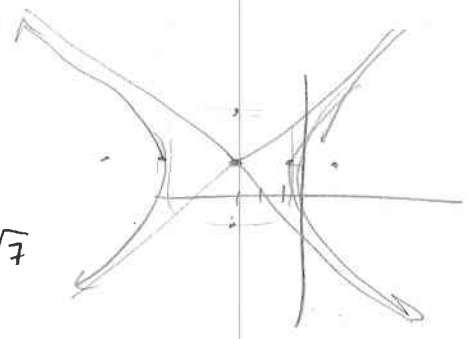
c=3√2

Center (-3, 1) transverse x

vertices (-3 ± √5, 1)

foci (-3 ± 3√2, 1)

asymptotes y-1 = ± √7/√5 (x+3)



O.  $\frac{x^2}{16} - \frac{(y-3)^2}{9} = 1$

a=4

c=5

b=3

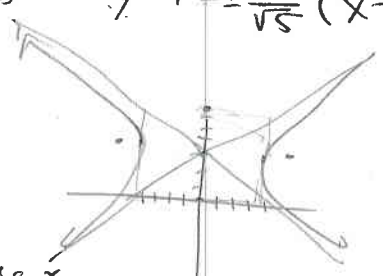
asymptotes y-3 = ± 3/4 (x)

transverse x

Center (0, 3)

vertices (4, 3), (-4, 3)

foci (-5, 3), (5, 3)



P. a=2 c=8

Center (1, 2)

vertices (1, 4), (1, 0)

foci (1, 10), (1, -6)

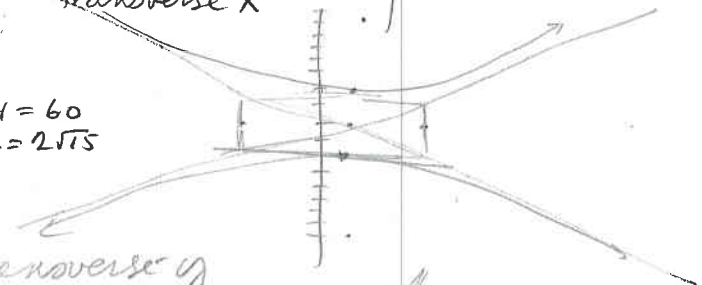
64-4=60  
b=2√15

transverse y

$\frac{(y-2)^2}{4} - \frac{(x-1)^2}{60} = 1$

asymptotes

y-2 = ± 1/30 (x-1)



Q. y-5 = ± 9/4 (x+3) transverse y

a=9 b=4

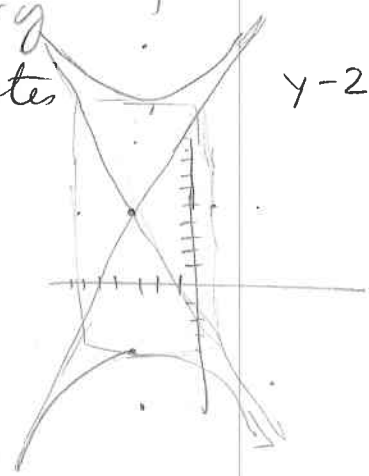
81+16=97 c=√97

Center (-3, 5)

vertices (-3, 14), (-3, -4)

foci (-3, 5 ± √97)

$\frac{(y-5)^2}{81} - \frac{(x+3)^2}{16} = 1$



# Hyperbolas, cont'd

r.  $y-5 = \pm \frac{9}{4}(x+3)$  transverse - x

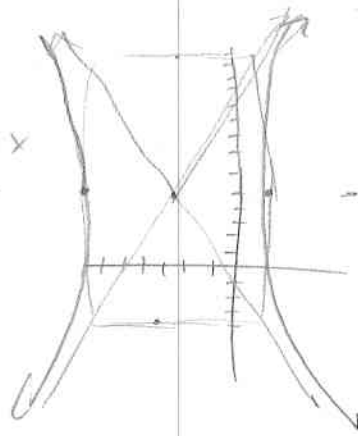
$\frac{b}{a} \Rightarrow b=9, a=4$

Center (3, 5)  $c = \sqrt{97}$

vertices (-7, 5) (1, 5)

foci  $(-3 \pm \sqrt{97}, 5)$

$$\frac{(x+3)^2}{16} - \frac{(y-5)^2}{81} = 1$$



s.  $9x^2 - 4(y^2 + 2y + 1) = 40 - 4$

$$\frac{9x^2}{36} - \frac{4(y+1)^2}{36} = \frac{36}{36} \Rightarrow \frac{x^2}{4} - \frac{(y+1)^2}{9} = 1$$

Center (0, -1)  $a=2, b=3$  transverse - x

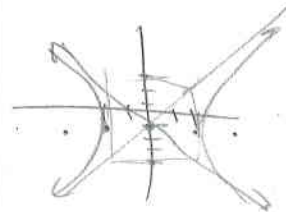
vertices  $(\pm 2, -1)$

$4+9=13$

$c = \sqrt{13}$

foci  $(\pm \sqrt{13}, -1)$

asymptotes  $y+1 = \pm \frac{3}{2}(x)$



t.  $-81x^2 + 25y^2 - 324x + 150y = 504$

$$25(y^2 + 6y + 9) - 81(x^2 + 4x + 4) = 504 + 225 - 324 = 405$$

$$\frac{(y+3)^2}{(9/81)} - \frac{(x+2)^2}{5} = 1$$

Center (-2, -3)

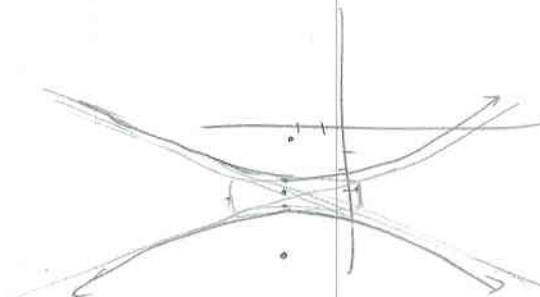
$a = \frac{\sqrt{5}}{9} \quad b = \sqrt{5} \quad c = \frac{\sqrt{410}}{9}$

$\frac{5}{81} + 5 = \frac{410}{81}$

vertices  $(-2, -3 \pm \frac{\sqrt{5}}{9})$

foci  $(-2, -3 \pm \frac{\sqrt{410}}{9})$

asymptotes  $y+3 = \pm \frac{1}{9}(x+2)$



## Overview:

a. ellipse

b. circle

Overview, cont'd.

c. parabola

d. hyperbola

e. parabola

f. circle

g. ellipse

h. circle

i. hyperbola.