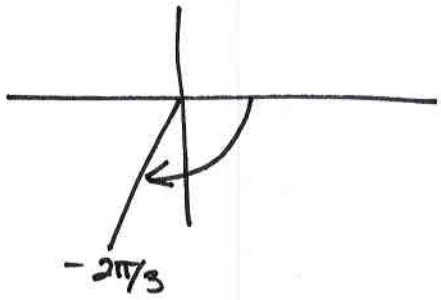
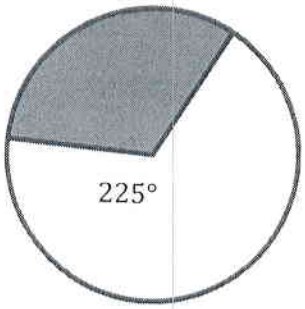
Math 1149, Exam #1, Fall 2013 Name KEY

Instructions: Show all work. Use exact values unless specifically asked to round.

1. Draw the angle $-2\pi/3$ in standard position. (4 points)



2. Find the area of the sector of the circle shown below, with a radius of 18 cm. (5 points)

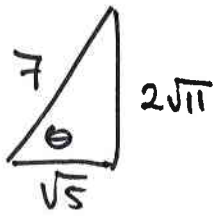


$225° \cdot \frac{\pi}{180°} = \frac{5\pi}{4}$

$A = \frac{1}{2}\left(\frac{5\pi}{4}\right)(18)^2 = \frac{405}{2}\pi$

$\approx 636.17 \text{ cm}$

3. Find the values of the other five trigonometric functions, given that $\cos\theta = \frac{\sqrt{5}}{7}$. (10 points)



$49 - 5 = 44$

$\sqrt{44} = \sqrt{4}\cdot\sqrt{11} = 2\sqrt{11}$

$\sin\theta = \frac{2\sqrt{11}}{7}$

$\cos\theta = \frac{\sqrt{5}}{7}$ given

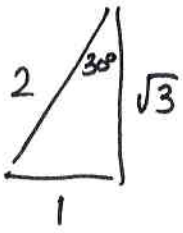
$\tan\theta = \frac{2\sqrt{11}}{\sqrt{5}} = \frac{2\sqrt{55}}{5}$

$\cot\theta = \frac{\sqrt{5}}{2\sqrt{11}} = \frac{\sqrt{55}}{22}$

$\sec\theta = \frac{7}{\sqrt{5}} = \frac{7\sqrt{5}}{5}$

$\csc\theta = \frac{7}{2\sqrt{11}} = \frac{7\sqrt{11}}{22}$

4. Find the value of the sine of 30°. (3 points)



$\sin 30° = \frac{1}{2}$

5. Simplify the expression $\sec 35° \csc 55° - \tan 35° \cot 55°$ (3 points)

$\sec 35° \cdot \sec 35° - \tan 35° \cdot \tan 35°$ cofunction identities

$\sec^2 35° - \tan^2 35°$

$1 + \tan^2 35° - \tan^2 35° = 1$

6. Find the exact value of the expressions. (4 points each)

a. $sec^2\left(\frac{\pi}{6}\right) - 4$

$\left(\frac{2}{\sqrt{3}}\right)^2 - 4 = \frac{4}{3} - 4 = -8/3$

b. $4 + tan^2\left(\frac{\pi}{3}\right)$

$4 + (\sqrt{3})^2 = 4 + 3 = 7$

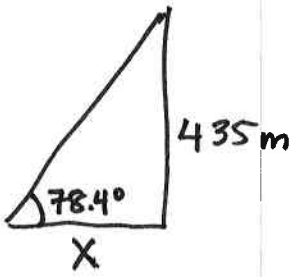
c. $1 + tan^2 30° - csc^2 45°$

$1 + \left(\frac{1}{\sqrt{3}}\right)^2 - (\sqrt{2})^2 = 1 + \frac{1}{3} - 2 = -2/3$

7. Use your calculator to approximate the value of $\cot\left(\frac{\pi}{18}\right)$. Round to 3 decimal places. (2 points)

$$\frac{1}{\tan(\pi/18)} \approx 5.671$$

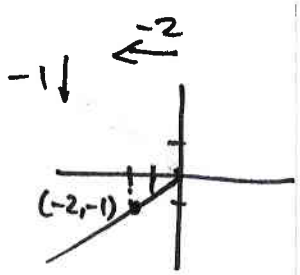
8. A guy wire is attached to the top of a 435m tower and to the ground a certain distance away. The angle that the wire makes with the ground is 78.4°. How far away from the center of the tower is the wire attached to the ground? (8 points)

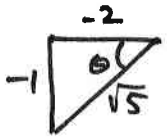


$$\tan 78.4° = \frac{435\text{ m}}{x}$$

$$x = \frac{435}{\tan 78.4°} \approx 89.29\text{ m}$$

9. The terminal side of an angle in standard position passes through the point (-2,-1). Find the values of the six trigonometric functions of the angle. (12 points)





$(-2)^2 + (-1)^2 = 4+1 = 5$

$r = \sqrt{5}$

$$\sin\theta = \frac{-1}{\sqrt{5}} = -\frac{\sqrt{5}}{5}$$

$$\cos\theta = \frac{-2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$$

$$\tan\theta = \frac{-1}{-2} = \frac{1}{2}$$

$$\cot\theta = 2$$

$$\sec\theta = -\frac{\sqrt{5}}{2}$$

$$\csc\theta = -\sqrt{5}$$

10. Find the reference angle for each of the given angles. (3 points each)

a. $\frac{5\pi}{4}$ $\pi/4$

b. $390°$ $30°$

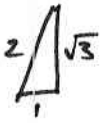
c. $-\frac{19\pi}{6}$ $\pi/6$

11. For each of the angles in Problem #10, state the value of the specified trigonometric function of the angle. Use the reference angle information above to obtain it. (3 points each)

a. $\cos\left(\frac{5\pi}{4}\right)$ $5\pi/4$ in QIII

$= -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$

b. $\tan(390°)$ $= \frac{1}{\sqrt{3}}$ in QI



c. $\csc\left(-\frac{19\pi}{6}\right)$ $= \csc(-7\pi/6)$ in QII

2

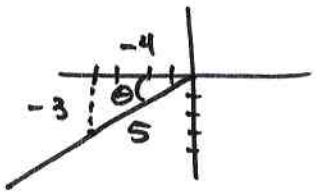
12. Use reference angles to find the exact value of $\tan 40° + \tan 140°$. You must show work. (3 points)

$\tan 140° = -\tan(40°)$

$\tan 40° + \tan 140° = \tan 40° - \tan 40° = 0$

13. Given that $\tan\theta = \frac{3}{4}$ and $\sin\theta < 0$, find the exact values of the remaining 5 trigonometric functions. Sketch a graph of the angle in standard position. (12 points)

tan + sin neg $\Rightarrow$ Q III



$\sin\theta = \frac{-3}{5}$

$\cos\theta = \frac{-4}{5}$

$\tan\theta = \frac{3}{4}$

$\cot\theta = \frac{4}{3}$

$\sec\theta = -\frac{5}{4}$

$\csc\theta = -\frac{5}{3}$

14. Answer each of the following questions. (3 points each).

a. For what values is $\tan\theta$ not defined?

odd multiples of $\pi/2$

$\theta = \frac{(2k+1)\pi}{2}$

b. What is the range of the cosine function?

$[-1, 1]$

c. Which of the six trigonometric functions are odd?

Sine, tangent, cotangent, cosecant

15. If $f(x) = \tan x$, and $f(a) = 2$, find the exact values of each of the following expressions. (3 points each).

a. $f(-a)$ $= -f(a) = -2$

tangent is odd

b. $f(a) + f(a + \pi)$ $=$

$f(a) + f(a) = 2 + 2 = 4$

tangent is π-periodic

c. $f(a - \pi)$ $=$

$f(a) = 2$

tangent is π-periodic