Math 2568, Quiz #11, Fall 2013 Name KEY

Instructions: Show all work. You may use a calculator, but you must show intermediate steps to receive full credit. Use exact answers unless specifically asked to round.

1. Given the bases $B = \left\{\begin{bmatrix}1\\1\\1\end{bmatrix}, \begin{bmatrix}1\\-1\\0\end{bmatrix}, \begin{bmatrix}1\\0\\-1\end{bmatrix}\right\}, C = \left\{\begin{bmatrix}2\\1\\0\end{bmatrix}, \begin{bmatrix}3\\1\\2\end{bmatrix}, \begin{bmatrix}1\\1\\-1\end{bmatrix}\right\}$, find the matrices $P_{C\leftarrow B}$ and $P_{B\leftarrow C}$. Then find the representation of the B-coordinate vector $[\vec{x}]_B = \begin{bmatrix}7\\5\\-4\end{bmatrix}$ in the basic C.

$$P_{C\leftarrow B} = P_C^{-1} P_B = \begin{bmatrix}-4 & 8 & 5\\ 2 & -3 & -2\\ 3 & -6 & -3\end{bmatrix}$$

$$P_{B\leftarrow C} = P_B^{-1} P_C = \begin{bmatrix}1 & 2 & 1/3\\ 0 & 1 & -2/3\\ 1 & 0 & 4/3\end{bmatrix}$$

$$P_{C\leftarrow B}[X]_B = \begin{bmatrix}-4 & 8 & 5\\ 2 & -3 & -2\\ 3 & -6 & -3\end{bmatrix}\begin{bmatrix}7\\5\\-4\end{bmatrix} = \begin{bmatrix}-8\\7\\3\end{bmatrix} = [\vec{x}]_C$$

2. Find the equilibrium (steady-state) vector of the following stochastic matrix algebraically:
$P = \begin{bmatrix}.7 & .05 & .1\\ .2 & .8 & .05\\ .1 & .15 & .85\end{bmatrix}$.

$$P - I = \begin{bmatrix}-.3 & .05 & .1\\ .2 & -.2 & .05\\ .1 & .15 & -.15\end{bmatrix} \text{ rref} \Rightarrow$$

$$\begin{bmatrix}1 & 0 & -.45\\ 0 & 1 & -.7\\ 0 & 0 & 0\end{bmatrix} \quad \begin{aligned} x_1 &= .45x_3\\ x_2 &= .7x_3\\ x_3 &= x_3\end{aligned} \Rightarrow \vec{x} = \begin{bmatrix}.45\\.7\\1\end{bmatrix} \Leftrightarrow \begin{bmatrix}9\\14\\20\end{bmatrix}$$

$9+14+20 = 43$

$$\vec{q} = \begin{bmatrix}9/43\\14/43\\20/43\end{bmatrix}$$ (checks in calc)

3. Give an example of a stochastic Markov chain matrix with an absorbing state.

$$\begin{bmatrix}.9 & .05 & 0\\ .05 & .9 & 0\\ .05 & .05 & 1\end{bmatrix}$$

answers will vary, but one column must be all zeros w/ one 1.