Instructions: Show all work. You must solve the problem by hand, but you may use your calculator to check the answer before submitting.

1. Solve the following system of equations by using an augmented matrix (reduced to at least echelon form, or reduced echelon form if the system is dependent). Write the answer as a column vector if independent, or in parametric form if dependent. If the system is inconsistent, be sure to state that result clearly.

$$\begin{cases} 2x_1 + x_2 - x_3 = -2 \\ 3x_2 + 4x_3 = 3 \\ 9x_1 - 4x_2 - 7x_3 = -9 \end{cases}$$

$$\begin{bmatrix} 2 & 1 & -1 & | & -2 \\ 0 & 3 & 4 & | & 3 \\ 9 & -4 & -7 & | & -9 \end{bmatrix} \quad \tfrac{1}{2}R_1 \to R_1 \quad \begin{bmatrix} 1 & \tfrac{1}{2} & -\tfrac{1}{2} & | & -1 \\ 0 & 3 & 4 & | & 3 \\ 9 & -4 & -7 & | & -9 \end{bmatrix} \quad -9R_1 + R_3 \to R_3$$

$$-9 \quad -\tfrac{9}{2} \quad \tfrac{9}{2} \quad 9$$

$$\begin{bmatrix} 1 & \tfrac{1}{2} & -\tfrac{1}{2} & | & -1 \\ 0 & 3 & 4 & | & 3 \\ 0 & -\tfrac{17}{2} & -\tfrac{5}{2} & | & 6 \end{bmatrix} \quad \tfrac{1}{3}R_2 \to R_2 \quad \begin{bmatrix} 1 & \tfrac{1}{2} & -\tfrac{1}{2} & | & -1 \\ 0 & 1 & \tfrac{4}{3} & | & 1 \\ 0 & -\tfrac{17}{2} & -\tfrac{5}{2} & | & 0 \end{bmatrix} \quad \tfrac{17}{2}R_2 + R_3 \to R_3$$

$$\tfrac{17}{2} \quad \tfrac{34}{3} \quad \tfrac{17}{2}$$

$$\begin{bmatrix} 1 & \tfrac{1}{2} & -\tfrac{1}{2} & | & -1 \\ 0 & 1 & \tfrac{4}{3} & | & 1 \\ 0 & 0 & \tfrac{53}{6} & | & \tfrac{17}{2} \end{bmatrix} \quad \tfrac{6}{53}R_3 \to R_3 \quad \begin{bmatrix} 1 & \tfrac{1}{2} & -\tfrac{1}{2} & | & -1 \\ 0 & 1 & \tfrac{4}{3} & | & 1 \\ 0 & 0 & 1 & | & \tfrac{51}{53} \end{bmatrix}$$

$$x_2 + \tfrac{4}{3}\left(\tfrac{51}{53}\right) = 1 \implies x_2 = -\tfrac{15}{53}$$

$$x_1 + \tfrac{1}{2}\left(-\tfrac{15}{53}\right) - \tfrac{1}{2}\left(\tfrac{51}{53}\right) = -1 \implies x_1 = -\tfrac{20}{53}$$

$$\vec{x} = \begin{bmatrix} -20/53 \\ -15/53 \\ 51/53 \end{bmatrix} \qquad \text{Consistent \& independent}$$

2. Give three examples of a 3x4 matrix in echelon form. If the matrix represents an augmented matrix, state whether each example represents a consistent or inconsistent system, and whether the system is dependent or independent (where appropriate).

$$\begin{bmatrix} 1 & * & * & | & * \\ 0 & 1 & * & | & * \\ 0 & 0 & 1 & | & * \end{bmatrix} \quad \text{consistent independent} \qquad \begin{bmatrix} 1 & * & * & | & * \\ 0 & 0 & 1 & | & * \\ 0 & 0 & 0 & | & 0 \end{bmatrix} \quad \text{consistent dependent}$$

$$\begin{bmatrix} 1 & * & * & | & * \\ 0 & 0 & 1 & | & * \\ 0 & 0 & 0 & | & 1 \end{bmatrix} \quad \text{inconsistent}$$

your answers may vary