Math 1116, Exam #1, Fall 2014 Name KEY

Instructions: Show all work. Answer each question as completely as possible. Use exact values. For counting problems you may use scientific notation (with three significant figures) for any numbers larger than a million.

1. How many license plates can you make if you are allowed nine digits total, with three capital letters (not including O), followed by three numbers, followed by three letters (not including O)? (5 points)

$25^3 \, 10^3 \, 25^3 = 2.44 \times 10^{11}$

2. Your remote garage door operator has a 4-digit code you need to enter to customize it. If you aren't allowed to repeat numbers, how many unique codes are there? (5 points)

$10P4 = 5040$

$10 \cdot 9 \cdot 8 \cdot 7 =$

3. There are 435 members of Congress. Suppose that on a particular day, ten members are allowed to speak in support of a particular bill. How many different ways can 10 members speak on any given day? (5 points)

$435C10 = 6 \times 10^{19}$

4. Suppose you want to rearrange the letters of 'caroline' to make a puzzle. How many different sequences of these letters are possible? (5 points)

$8P8 = 8! = 40,320$

5. There are four prizes of differing values in a raffle drawing with 100 tickets sold. How many different ways can you distribute the four prizes? (5 points)

$100P4 = 94,109,400$

6. A football team designates three players to bring in the balls, carry the miscellaneous equipment, and hand out the water bottles before and after practice. If there are 14 players on the team, how many ways can the three jobs at each practice? (5 points)

$14P3 = 2184$

7. A math club has 15 members and they want to form a three-member committee to plan an event. In how many different ways can the committee be chosen? (5 points)

$15C3 = 455$

8. A coin is flipped 12 times. In how many different ways can the coin come up heads 5 times? (5 points)

$12C5 = 792$

9. If I choose a 5-card poker hand from a standard deck of cards, how many different hands are possible if it contains only spades? (5 points)

$13C5 = 1287$

10. An election was cast and resulted in the following preference table.

Place/# of Votes	18	12	10	9	4	2
1st	A	B	C	D	E	E
2nd	D	E	B	C	B	C
3rd	E	D	E	E	D	D
4th	C	C	D	B	C	B
5th	B	A	A	A	A	A

a. Determine the winner of the election using the plurality method. (7 points)

55 total votes
28 for majority

A = 18
B = 12
C = 10
D = 9
E = 6

A wins

b. Determine the winner of the election using the plurality with elimination method/instant run-off voting. (8 points)

A = 18 B = 12 C = 10 D = 9 ~~E = 6~~

A = 18 B = 12+4 = 16 C = 10+2 = 12 ~~D = 9~~

A = 18 ~~B = 16~~ C = 12+9 = 21

C = 21+4+12 = 37

C wins

A = 18

c. Determine the winner of the election using the Borda Count method. (8 points)

$A = 5(18) + 1(12+10+9+4+2) = 127$

$B = 1(18) + 5(12) + 4(10) + 2(9) + 4(4) + 2(2) = 156$

$C = 2(18) + 2(12) + 5(10) + 4(9) + 2(4) + 4(2) = 162$

$D = 4(18) + 3(12) + 2(10) + 5(9) + 3(4) + 3(2) = 191$ ← Dunis

$E = 3(18) + 4(12) + 3(10) + 3(9) + 5(4) + 5(2) = 189$

d. Determine the winner using the method of pairwise comparisons. (8 points)

A-B 18 vs 12+10+9+4+2

A-C 18 vs 12+10+9+4+2

A-D 18 vs 12+10+9+4+2

A-E 18 vs 12+10+9+4+2

B-C 12+9 vs 18+10+9+2

B-D 12+10+9 vs 18+9+2

B-E 12+10 vs 18+9+4+2

C-D 10+2 vs 18+12+9+4

C-E 10+9 vs 18+12+4+2

D-E 18+9 vs 12+10+9+2

A =

B = 1

C = 11

D = 111

E = 1111 ← Ewins

e. Who earned 2nd, 3rd, 4th and 5th place using the extended ranking of the method of pairwise comparisons? (7 points)

E - 1st

D - 2nd

C - 3rd

B - 4th

A - 5th

f. Is there a majority criterion violation? Why or why not? (5 points)

No, there is no majority criterion violations since no one has a majority of first place votes

g. Is there a Condorcet criterion violation? Why or Why not? (5 points)

yes. Plurality, Plurality w/ Elimination and Borda Count all have a Condorcet violation since they all disagree w/ pairwise winner.

h. Is there an independence of irrelevant alternative criterion violation? Why or why not? (5 points)

yes. the plurality method has an IIA violation since it disagrees w/ the results of dropping losing candidates from race (Plurality w/ Elimination)

11. What would be required to get a monotonicity criterion violation? (6 points)

after a count of the vote determines a winner a group of voters votes strategically to give winning candidate more votes, but in doing so makes the previous winner end up losing the recounted vote

12. Consider the weighted voting system [15: 11, 7, 5, 2].
    a. Are there any dictators? Dummies? (5 points)

no dictators, P4 is a dummy

    b. Does anyone have veto power? (4 points)

P1 has a veto

    c. List all the winning coalitions. (8 points)

{P1, P2, P3, P4}
{P1, P2, P3}
{P1, P2, P4}
{P1, P3, P4}
{P1, P2}
{P1, P3}

    d. Underline the critical players in each coalition above and use that information to calculate the Banzhaf power index. (8 points)

P1: $\frac{6}{10} = 60\%$

P2: $\frac{2}{10} = 20\%$

P3 = $\frac{2}{10} = 20\%$

P4 = $\frac{0}{10} = 0\%$

e. List all sequential coalitions. (8 points)

<P1, P2, P3, P4>
<P1, P2, P4, P3>
<P1, P3, P2, P4>
<P1, P3, P4, P2>
<P1, P4, P2, P3>
<P1, P4, P3, P2>
<P2, P1, P3, P4>
<P2, P1, P4, P3>
<P2, P1, P3, P4>
<P2, P3, P4, P1>
<P2, P4, P1, P3>
<P2, P4, P3, P1>
<P3, P1, P2, P4>
<P3, P1, P4, P2>
<P3, P2, P1, P4>
<P3, P2, P4, P1>
<P3, P4, P1, P2>
<P3, P4, P2, P1>
<P4, P1, P2, P3>
<P4, P1, P3, P2>
<P4, P2, P1, P3>
<P4, P2, P3, P1>
<P4, P3, P1, P2>
<P4, P3, P2, P1>

f. Use the information above and underline all pivotal players. Then use that information to calculate the Shapley-Shubik Power Index. (8 points)

$P1 = \frac{16}{24} = 67\%$ $P3 = \frac{4}{24} = 17\%$

$P2 = \frac{4}{24} = 17\%$ $P4 = \frac{0}{24} = 0\%$

g. How does the power calculation compare to the proportion of total votes each voter holds. Is there any evidence that one of the players has substantially more power than the weight of their votes implies? (8 points)

$P1 = \frac{11}{25} = 44\%$ $P4 = \frac{2}{25} = 8\%$

$P2 = \frac{7}{25} = 28\%$

$P3 = \frac{5}{25} = 20\%$

P1 has more power than they should and others have less.

13. Consider the weighted voting system $[q: 25, 21, 18, 16, 12, 5]$.

a. What is the smallest value that the quota can take? (4 points)

$\frac{97}{2} = 48.5$ **49**

b. What is the largest possible value of the quota? (4 points)

97

c. What would the value of the quota be if 2/3 of the vote was needed for passage? (4 points)

$97 \cdot \frac{2}{3} = 64.\overline{6}$ **65**

14. Calculate the following values. Show as much simplifying work as possible, then use your calculator to compute the final result. (5 points each)

a. $P(13,4)$

13P4 = 17,160 $\quad \frac{13!}{9!} = 13 \cdot 12 \cdot 11 \cdot 10$

b. 9!

362,880 $\quad 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 =$

c. $C(20,7)$

20C7 = 77,520 $\quad \frac{20!}{7!\,13!} = \frac{20 \cdot 19 \cdot \overset{3}{18} \cdot 17 \cdot \overset{4}{16} \cdot 15 \cdot 14}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7}$

d. $\binom{16}{10}$

16C10 = 8008 $\quad \frac{16!}{10!\,6!} = \frac{\overset{4}{16} \cdot 15 \cdot 14 \cdot 13 \cdot 12 \cdot 11}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6} =$

15. How many possible coalitions are there (winning and losing both) in a weighted voting system with 6 players. (6 points)

$6C6 + 6C5 + 6C4 + 6C3 + 6C2 + 6C1 =$

$1 + 6 + 15 + 20 + 15 + 6 = 63$

16. How many possible sequential coalitions are there in a weighted voting system with 8 players? (6 points)

$8! = 8P8 = 40,320$

17. In the table below is a set of preference ballots for an election. Convert the raw ballots into a preference table. Fill in the second table with the results. (10 points)

A	B	B	A	C	A	B	B	A	B	A	A	C	C	B
B	C	A	C	B	B	C	C	B	A	C	B	B	B	C
C	A	C	B	A	C	A	A	C	C	B	C	A	A	A

	4	2	4	2	3	0
1st	A	A	B	B	C	C
2nd	B	C	C	A	B	A
3rd	C	B	A	C	A	B

18. What is Arrow's Impossibility Theorem? Describe it. Why is it important? (6 points)

This theorem says that there is no perfectly fair voting system. All systems will sometimes violate one or more fairness criteria. The best we can hope to do is choose which criteria could be violated and which won't be.

19. What is strategic voting? When is it employed and why is it a possible problem in an election? (7 points)

Strategic voting is when a voter casts a ballot for a candidate they don't really prefer, usually in an effort to keep a candidate they prefer even less from winning

it can be a problem because it tends to reduce multi-party systems to 2-party systems (in plurality voting); and can lead to fairness problems in other voting systems (like plurality w/ elimination).

Also, voter's preferences are not really well reflected in the results of an election.