Math 1116, Exam #2, Fall 2014 Name KEY

Instructions: Show all work. Answer each question as completely as possible. Use exact values. For counting problems you may use scientific notation (with three significant figures) for any numbers larger than a million. You may round decimals to three significant figures as well.

1. Consider a state license plate with six characters total, with three capital letters (not including O), followed by three numbers. What is the probability of a random plate having only vowels and even numbers? (8 points)

no "O"

$$\frac{4^3 \cdot 5^3}{25^3 \cdot 10^3} = 5.12 \times 10^{-4}$$

2. There are 435 members of Congress. Suppose that on a particular day, ten members are allowed to speak in support of a particular bill. What is the probability that a random member of Congress will be chosen to speak? (8 points)

$$\frac{10}{435} = .0229885$$

3. There are four prizes of differing values in a raffle drawing with 100 tickets sold. What is the probability that you will win one of the prizes? (8 points)

$$\frac{4}{100} = 4\%$$

4. A math club has 15 members (9 men and 6 women) and they want to form a three-member committee to plan an event. What is the probability that the committee will consist of all women? (8 points)

$$\frac{\binom{6}{3}}{\binom{15}{3}} = .043956$$

5. A coin is flipped 12 times. What is the probability that the coin comes up heads 5 times? (8 points)

$$\frac{\binom{12}{5}}{2^{12}} = .193359$$

6. If I choose a 5-card poker hand from a standard deck of cards, what is the probability it will contain only spades? (8 points)

$$\frac{\binom{13}{5}}{\binom{52}{5}} = 4.95 \times 10^{-4}$$

7. Apportion 250 seats among the states below using Hamilton's Method. Complete the table. (16 points)

State	Population	Standard Quota	Lower Quota	Upper Quota	+1	Final Apportionment
Callari	87,912	94.202	94	95		94
Dardia	55,265	59.219	59	60		59
Ellalia	48,975	52.479	52	53	+1	53
Phimineia	24,035	25.755	25	26	+1	26
Q'ndar	17,119	18.344	18	19		18
Total	233,306		248			250
Standard Divisor	933.224					

8. Apportion 25 seats among the states below using Jefferson's Method. Complete the table. (17 points)

State	Population	Standard Quota	MQ	MQ	Final Apportionment
Meniad	403	7.001	7.75	7.589	7
Noozien	318	5.52	6.1153	5.988	5
Oraq	186	3.23	3.5769	3.50	3
Rhythine	532	9.24	10.23	10.0188	10
Total	1439	24			25
Standard Divisor	57.56	→	52	→ 53.1	

Does this apportionment produce a Quota Rule violation? (3 points)

NO

9. Apportion 405 seats among the states below using Huntington-Hill's Method. Complete the Table. (16 points)

State	Population	Standard Quota	Lower Quota	Upper Quota	$\sqrt{LQ \times UQ}$	Final Apportionment
Hunta	48,400	93.34286	93	94	93.49866	93
Iobian	73,400	141.55714	141	142	141.49911	142
Jindi	29,700	57.27857	57	58	57.4978	57
Kelerna	15,300	29.5071	29	30	29.4957	30
Laria	43,200	83.3142	83	84	83.4985	83
Total	210,000					405
Standard Divisor	518.5185					

10. Determine which paradox is illustrated below and indicate exactly why (and where) the paradox is evident. Show any and all computations needed to support your conclusion. (7 points each)

a.

$$\text{Standard divisor} = \frac{10000}{100} = 100$$

Branch	A	B	Total
Employees	1045	8955	10,000
Standard Quota	10.45	89.55	100
Lower Quota	10	89	99
Hamilton's	10	90	100

$$\text{Standard divisor} = \frac{10525}{105} = 100.24$$

Branch	A	B	C	Total
Employees	1045	8955	525	10,525
Standard Quota	10.42	89.34	5.24	105
Lower Quota	10	89	5	104
Hamilton's	11	89	5	105

New States' Paradox

b.

	Fall Semester			Spring Semester		
Subject	Number Fall	SQ	Positions	Number Spring	New SQ	Positions
Math	476	9.5180	10	484	9.6338	9
English	975	19.4961	19	990	19.7054	20
Science	3550	70.9858	71	3550	70.6608	71
Total	5001		100	5024		100

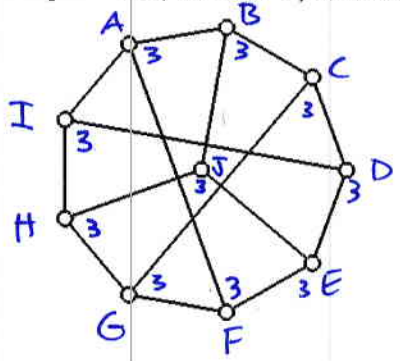
$\frac{484-476}{476} = .0168$

$\frac{990-975}{975} = .0153$ slower growth

Population paradox

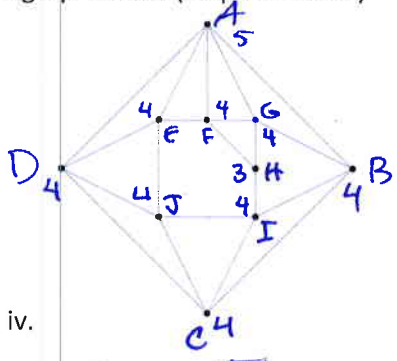
11. For each of the graphs below, determine the following: a) label the vertices if they do not already have labels, b) state the degree of each vertex, c) is the graph connected? d) Does the graph have any bridges? If so, list one. e) How many edges does the graph have? (12 points each)

i.



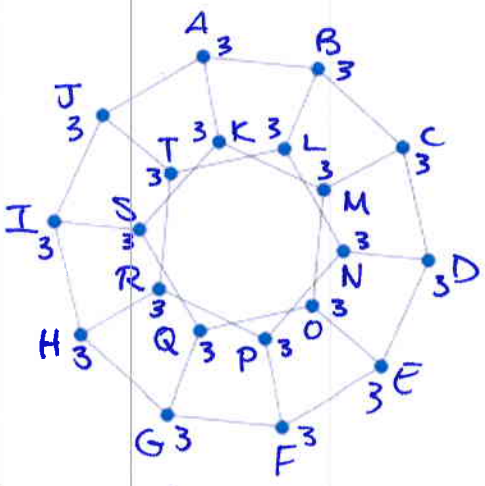
Connected
no bridges
15 edges

iv.



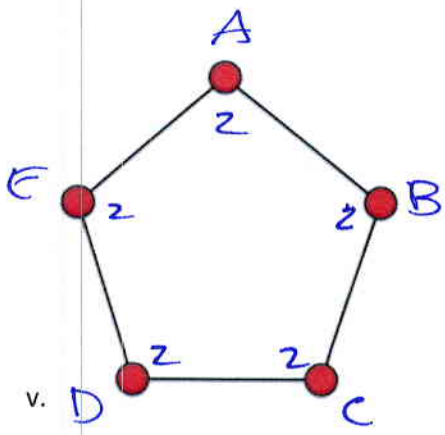
Connected
no bridges
20 edges

ii.



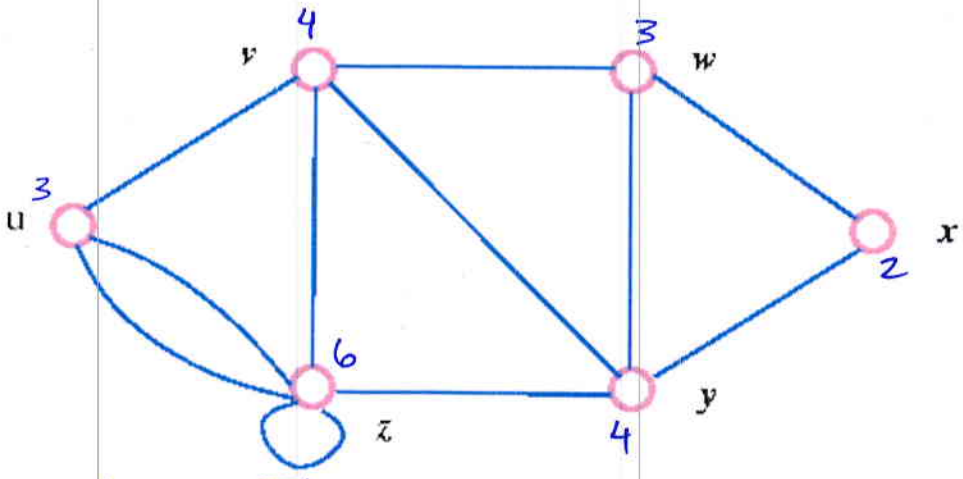
Connected
no bridges
30 edges

v.



Connected
no bridges
5 edges

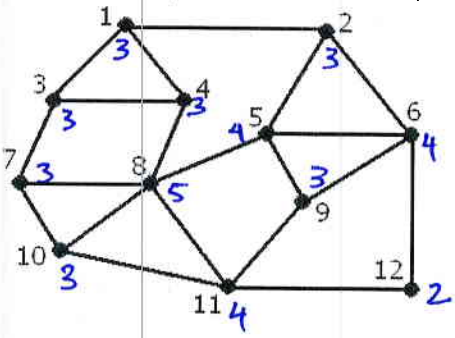
iii.



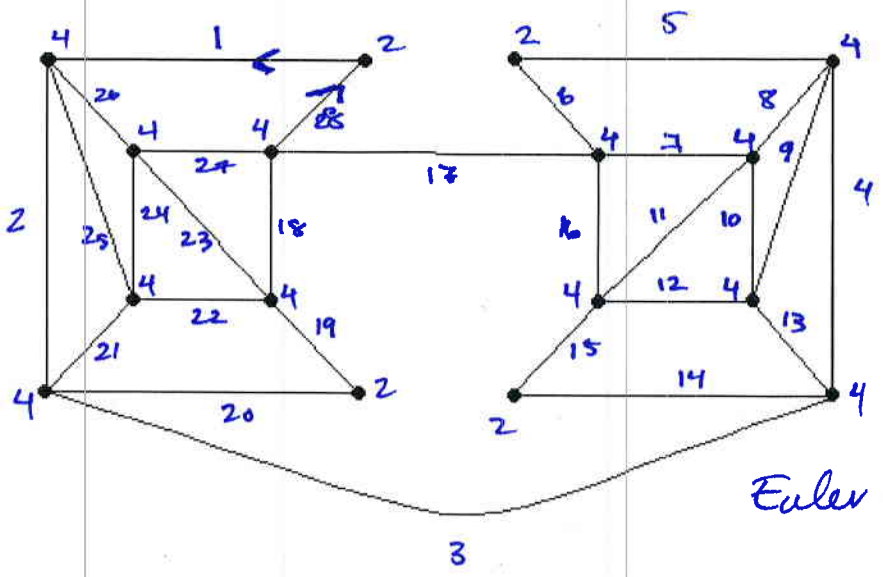
Connected
no bridges
11 edges

12. For each of the graphs below, determine if it has an Euler circuit. If so, find one. Label the edges in order you traverse them. If not, does it have an Euler path? If so, find one. If it doesn't have either, explain why not. (8 points each)

a.

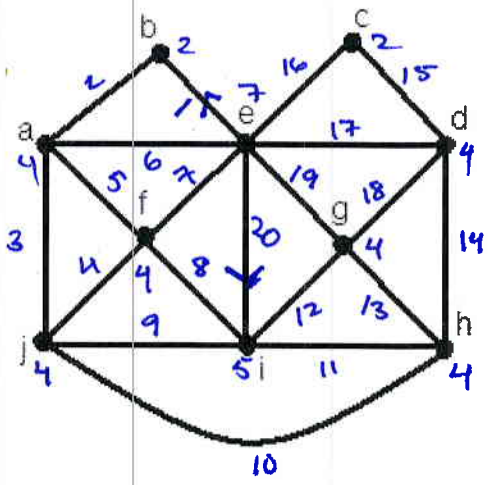


no circuit or path
too many odd vertices



Euler circuit

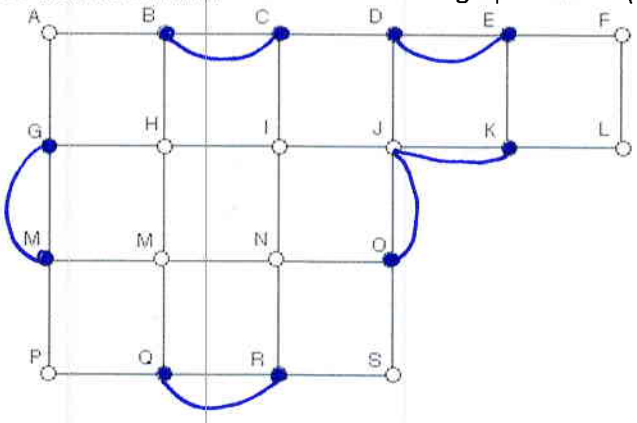
b.



Euler path

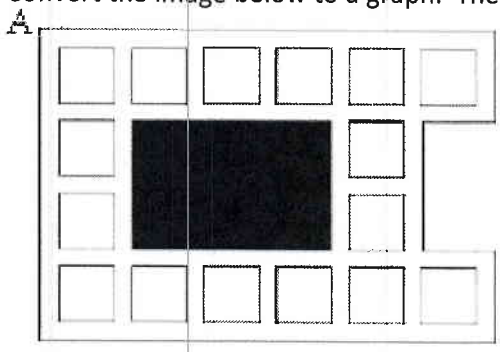
c.

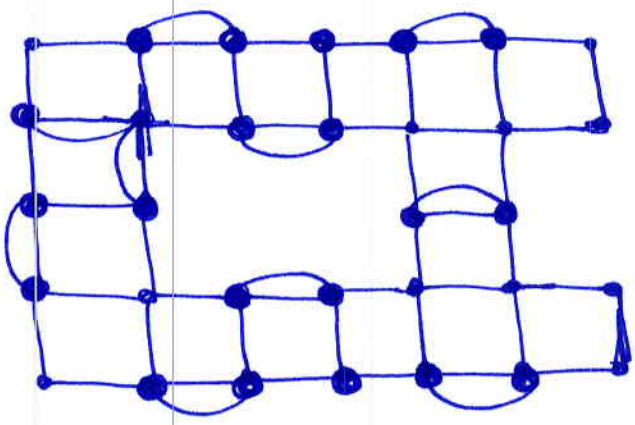
13. Find the minimal eulerization of the graph below. (10 points)



answers may vary but no more than 6 new edges

14. Convert the image below to a graph. Then find an optimal semi-eulerization. (12 points)





10 is a minimum