Math 1151, Area Under a Curve Activity, Fall 2014 Name KEY

Instructions: For each of the functions below, sketch the region on the indicated interval, and find the area under the curve using the right endpoint limit method. Approximate the area for $n = 6$, and also find the limit as $n \to \infty$.

1. $f(x) = -4x + 5, [0,1]$

$\Delta x = \frac{1}{n}$

$x_i = 0 + \frac{i}{n} = \frac{i}{n}$

$$\sum\left(-4\left(\frac{i}{n}\right)+5\right)\Delta x = \sum\left(\frac{-4i}{n} + 5\right)\frac{1}{n} = \sum\left(\frac{-4i}{n^2} + \frac{5}{n}\right)$$

$$\frac{-4}{n^2}\sum i + \sum\left(\frac{5}{n}\right) = \frac{-4}{n^2}\left(\frac{n(n+1)}{2}\right) + \frac{5}{n}\cdot n =$$

$$\frac{-4n-4}{2n} + 5 = \frac{-4n}{2n} - \frac{4}{2n} + 5 = -2 - \frac{4}{2n} + 5 = 3 - \frac{4}{2n}$$

$$\lim_{n\to\infty} 3 - \frac{4}{2n} = 3$$

2. $f(x) = x^2 + 1, [0,3]$

$\Delta x = \frac{3}{n}$ $\quad x_i = 0 + \frac{3i}{n} = \frac{3i}{n}$

$$\sum\left[\left(\frac{3i}{n}\right)^2 + 1\right]\frac{3}{n} = \sum\left(\frac{9i^2}{n^2} + 1\right)\frac{3}{n} = \sum\left(\frac{27i^2}{n^3} + \frac{3}{n}\right) =$$

$$\frac{27}{n^3}\sum i^2 + \sum\frac{3}{n} = \frac{\overset{9}{\cancel{27}}}{n^{\cancel{3}\,2}}\cdot\frac{\cancel{n}(n+1)(2n+1)}{\underset{2}{\cancel{6}}} + \frac{3}{n}\cdot n =$$

$$\frac{9}{2n^2}(2n^2+3n+1) + 3 = \frac{18n^2 + 27n + 9}{2n^2} + 3 = 9 + \frac{27}{2n} + \frac{9}{2n^2} + 3 =$$

$$\lim_{n\to\infty} 12 + \frac{27}{2n} + \frac{9}{2n^2} = 12$$

3. $f(x) = 2x - 5, [3,7]$ $\Delta x = \frac{4}{n}$ $x_i = 3 + \frac{4i}{n}$

$$\sum \left[2\left(3 + \frac{4i}{n}\right) - 5\right]\frac{4}{n} = \sum \left(6 + \frac{8i}{n} - 5\right)\frac{4}{n} = \sum \left(1 + \frac{8i}{n}\right)\frac{4}{n} =$$

$$\sum\left(\frac{4}{n} + \frac{32i}{n^2}\right) = \sum \frac{4}{n}\ \frac{32}{n^2}\sum i = \frac{4}{\cancel{n}}\cancel{n} + \frac{\overset{16}{\cancel{32}}}{n^{\cancel{2}}}\ \frac{\cancel{n}(n+1)}{\cancel{2}} =$$

$$4 + \frac{16\cancel{n}}{\cancel{n}} + \frac{16}{n} = 20 + \frac{16}{n} \qquad \lim_{n\to\infty} 20 + \frac{16}{n} = 20$$

4. $f(x) = 27 - x^3, [1,3]$ $\Delta x = \frac{2}{n}$ $x_i = 1 + \frac{2i}{n}$

$$\sum\left(27 - \left(1 + \frac{2i}{n}\right)^3\right)\frac{2}{n} = \sum\left(27 - \left(1 + \frac{6i}{n} + \frac{12i^2}{n^2} + \frac{8i^3}{n^3}\right)\right)\frac{2}{n} =$$

$$\sum\left(26 - \frac{6i}{n} - \frac{12i^2}{n^2} - \frac{8i^3}{n^3}\right)\frac{2}{n} = \sum\left(\frac{52}{n} - \frac{12i}{n^2} - \frac{24i^2}{n^3} - \frac{16i^3}{n^4}\right)$$

$$= \sum\frac{52}{n} - \frac{12}{n^2}\sum i - \frac{24}{n^3}\sum i^2 - \frac{16}{n^4}\sum i^3 =$$

$$\frac{52}{\cancel{n}}\cancel{n} - \frac{\overset{6}{\cancel{12}}}{n^{\cancel{2}}}\ \frac{\cancel{n}(n+1)}{\cancel{2}} - \frac{\overset{4}{\cancel{24}}}{n^{\cancel{3}}{}_2}\ \frac{\cancel{n}(n+1)(2n+1)}{\cancel{6}} - \frac{\overset{4}{\cancel{16}}}{n^{\cancel{4}}{}_2}\ \frac{\cancel{n^2}(n+1)^2}{\cancel{4}}$$

$$52 - \frac{6\cancel{n}}{\cancel{n}} - \frac{6}{n} - \frac{8\cancel{n^2}}{\cancel{n^2}} - \frac{12\cancel{n}}{n^{\cancel{2}}} - \frac{4}{n^2} - \frac{4\cancel{n^2}}{\cancel{n^2}} - \frac{8\cancel{n}}{n^{\cancel{2}}} - \frac{4}{n^2}$$

$$= \lim_{n\to\infty} 52 - 6 - \frac{6}{n} - 8 - \frac{12}{n} - \frac{4}{n^2} - 4 - \frac{8}{n} - \frac{4}{n^2} =$$

$$52 - 6 - 8 - 4 = 34$$