Math 1151, Exam #2, Fall 2014 Name KEY

Instructions: Show all work. Answers without work may only receive partial credit. If you are asked for an explanation, explain as completely as possible. Use exact answers unless specifically asked to round.

1. Find the derivative of the following functions. (5 points each)
   a. $f(x) = \sec x \tan x$

$$f'(x) = (\sec x \tan x)\tan x + \sec x(\sec^2 x) =$$
$$\sec x \tan^2 x + \sec^3 x$$

   b. $y = e^{-x} \sin x$

$$y' = -e^{-x}\sin x + e^{-x}\cos x = e^{-x}(\cos x - \sin x)$$

   c. $g(x) = (3x^2 + 7x)^{10}$

$$g'(x) = 10(3x^2+7x)^9(6x+7)$$

   d. $h(t) = e^{\tan t}$

$$h'(t) = e^{\tan t}\sec^2 t$$

e. $y = e^{x^2+1}\sin(x^3)$

$$y' = e^{x^2+1}(2x)\sin(x^3) + e^{x^2+1}\cos(x^3)3x^2$$

$$= e^{x^2+1}x(2\sin x^3 + 3x\cos(x^3))$$

f. $p(x) = \cos^4(e^{3x}+1)$

$$p'(x) = 4\cos^3(e^{3x}+1)(-\sin(e^{3x}+1))(e^{3x}\cdot 3)$$

$$= -12e^{3x}\cos^3(e^{3x}+1)\sin(e^{3x}+1)$$

g. $r(s) = \ln(\ln(s))$

$$r'(s) = \frac{1}{\ln s}\cdot\frac{1}{s} = \frac{1}{s\ln s}$$

h. $q(t) = 5^{3t}$

$$q'(t) = (\ln 5)5^{3t}\cdot 3 = (3\ln 5)5^{3t}$$

i. $y = \sin^{-1}(e^{\sin x})$

$$y' = \frac{1}{\sqrt{1-(e^{\sin x})^2}}\cdot e^{\sin x}\cdot\cos x = \frac{\cos x\, e^{\sin x}}{\sqrt{1-e^{2\sin x}}}$$

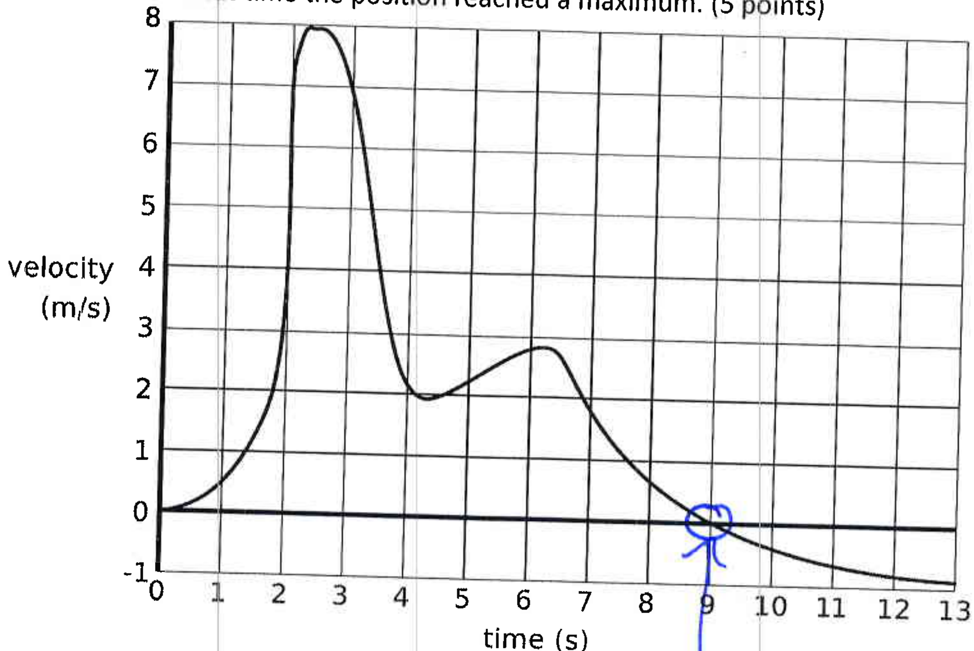
j. $y = \tan^{-1}(2x^2 - 4)$

$$\frac{1}{1+(2x^2-4)^2} \cdot (4x) = \frac{4x}{1+(2x^2-4)^2}$$

k. $l(x) = \frac{1}{\tan^{-1}(x^2+4)}$ $= \left[\tan^{-1}(x^2+4)\right]^{-1}$

$$-1\frac{1}{\left[\tan^{-1}(x^2+4)\right]^{-2}} \cdot \frac{1}{1+(x^2+4)^2} \cdot 2x$$

2. Shown below is the graph of the velocity of a function relative to time. Use this graph to determine at what time the position reached a maximum. (5 points)



derivative is zero here

t=9 is when it reached a max

reached a max

7. Find the critical points and inflection points of the function $y = x^2e^{-x}$. Develop sign charts for both the first and second derivatives and use that information to determine whether each critical point is a maximum or a minimum. (13 points)

$f'(x) = 2xe^{-x} + x^2e^{-x}(-1)$ $\quad e^{-x} \cdot x(2-x) = 0$

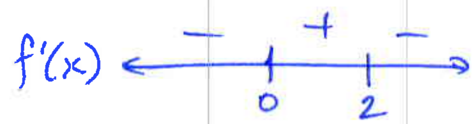
$e^{-x}(2x - x^2)$ $\quad x = 0, \; x = 2$

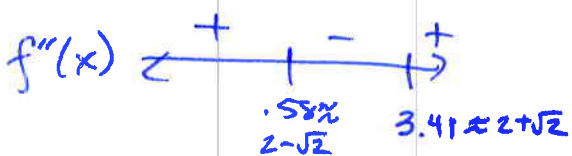
$f''(x) = -e^{-x}(2x-x^2) + e^{-x}(2-2x) = e^{-x}(2-2x-2x+x^2)$

$= e^{-x}(2-4x+x^2)$

$\frac{4 \pm \sqrt{16-4(2)}}{2} = \frac{4 \pm \sqrt{8}}{2} =$

$2 \pm \sqrt{2}$





x=0

graph is concave up ∪

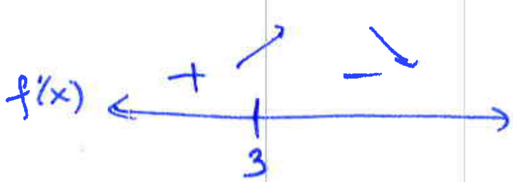
so this is a minimum

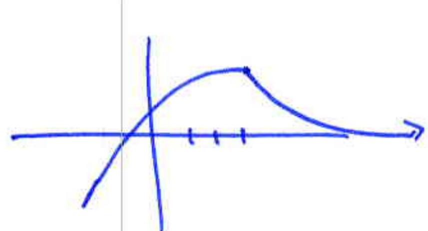
x=2

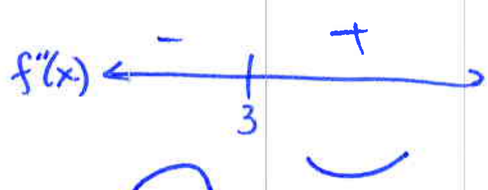
graph is concave down ∩

so this is a maximum

8. Sketch a curve with the following properties: $f' > 0$ and $f'' < 0$ for $x < 3$; and $f' < 0$ and $f'' > 0$ for $x > 3$. (7 points)







3. Find the derivative $\frac{dy}{dx}$ for the implicit function $e^{2y} + y^2 - x^3y + 6x^7 = \cos x$. (8 points)

$$e^{2y} \cdot 2y' + 2yy' - 3x^2y - x^3y' + 42x^6 = -\sin x$$

$$y'\left[2e^{2y} + 2y - x^3\right] = 3x^2y - \sin x - 42x^6$$

$$y' = \frac{dy}{dx} = \frac{3x^2y - \sin x - 42x^6}{2e^{2y} + 2y - x^3}$$

4. Use logarithmic differentiation to find the derivatives of the following functions. (8 points each)

a. $y = (\sin x)^{\ln x}$

$$\ln y = \ln\left[(\sin x)^{\ln x}\right] = (\ln x)(\ln \sin x)$$

$$\frac{1}{y}y' = \frac{1}{x}\ln \sin x + \ln x \cdot \frac{1}{\sin x} \cdot \cos x =$$

$$y' = \left[\frac{\ln \sin x}{x} + \cot x \cdot \ln x\right](\sin x)^{\ln x}$$

b. $f(x) = \frac{(x+1)^{3/2}(x-4)^{5/2}}{(5x+3)^{2/3}}$

$$\ln y = \ln\left[\frac{(x+1)^{3/2}(x-4)^{5/2}}{(5x+3)^{2/3}}\right]$$

$$\ln y = \frac{3}{2}\ln(x+1) + \frac{5}{2}\ln(x-4) - \frac{2}{3}\ln(5x+3)$$

$$\frac{1}{y}y' = \frac{3}{2} \cdot \frac{1}{x+1} + \frac{5}{2} \cdot \frac{1}{x-4} - \frac{2}{3} \cdot \frac{1}{5x+3} \cdot 5 \Rightarrow$$

$$y' = \left[\frac{3}{2(x+1)} + \frac{5}{2(x-4)} - \frac{10}{3(5x+3)}\right]\frac{(x+1)^{3/2}(x-4)^{5/2}}{(5x+3)^{2/3}}$$

9. Graph the function $f(x)=\frac{\ln x}{x^2}$ on its domain. Use properties of derivatives to create the graph. (You may check your results in your calculator, but you will be graded on the work you show to support your graph.) (16 points)

$x>0$ $\qquad f(x)=\ln x\cdot x^{-2}$

$$f'(x)=\frac{1}{x}\cdot x^{-2}+\ln x\cdot(-2x^{-3})=\frac{1}{x^3}-\frac{2\ln x}{x^3}$$

$$=\frac{1-2\ln x}{x^3}$$

Critical points $(x=0)$

$1-2\ln x=0\Rightarrow 2\ln x=1\Rightarrow \ln x=\frac{1}{2}\Rightarrow$

$e^{1/2}=x=\sqrt{e}$

$\approx 1.6487\ldots$

$$f''(x)=-\frac{2}{x}\cdot\frac{1}{x^3}-(1-2\ln x)(-3)\frac{1}{x^4}$$

$$=\frac{1}{x^4}\left[-2-(1-2\ln x)(-3)\right]=\frac{1}{x^4}\left[-2+3(1-2\ln x)\right]=$$

$$\frac{1}{x^4}\left[-2+3-6\ln x\right]=\frac{1}{x^4}\left[1-6\ln x\right]=0$$

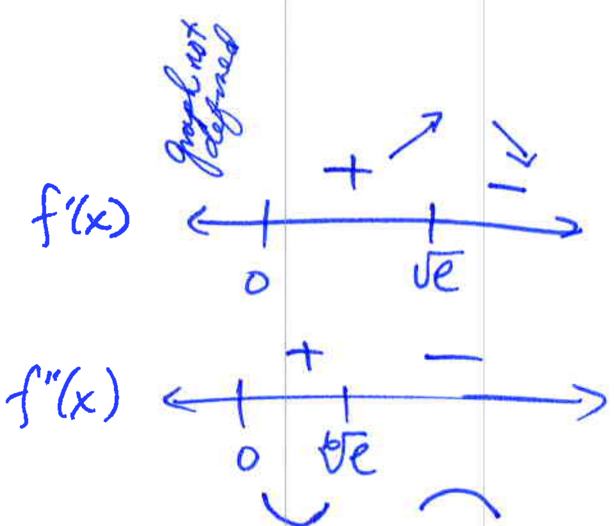
inflection points $(x=0)$

$1-6\ln x=0$

$6\ln x=1$

$\ln x=1/6$

$x=e^{1/6}=\sqrt[6]{e}\approx 1.18136\ldots$



$f(\sqrt{e})$ is a max