Math 1151, Quiz #11, Fall 2014 Name KEY

Instructions: Show all work. Answers without work may only receive partial credit. If you are asked for an explanation, explain as completely as possible. Use exact answers unless specifically asked to round.

1. Use L'Hopital's Rule to find the limits. Be sure the function is in the appropriate indeterminate form to apply the rule or perform the necessary algebra to correct the form.

   a. $\lim_{x \to 0} \frac{\tan 7x}{\sin 4x}$ $= \lim_{x \to 0} \frac{7\sec^2 7x}{4\cos 4x} = \frac{7}{4}$

   b. $\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^{\ln x} = L$ $\ln L = \lim_{x \to \infty} \ln\left(1 + \frac{1}{x}\right) \ln x = \lim_{x \to \infty} \frac{\ln\left(1 + \frac{1}{x}\right)}{(\ln x)^{-1}} =$

   $\lim_{x \to \infty} \frac{\frac{1}{1+\frac{1}{x}}(-1x^{-2})}{-\frac{1}{(\ln x)^2} \cdot \frac{1}{x}} \cdot \frac{x^2}{x^2} = \lim_{x \to \infty} \frac{\frac{1}{1+\frac{1}{x}}}{\frac{x}{(\ln x)^2}} = \lim_{x \to \infty} (\ln x)^2 \cdot \frac{1}{1+\frac{1}{x}} \cdot \frac{1}{x} =$

   $\lim_{x \to \infty} (\ln x)^2 \cdot \frac{1}{1+x} = \lim_{x \to \infty} \frac{2\ln x \cdot \frac{1}{x}}{1} = \lim_{x \to \infty} \frac{2 \cdot \frac{1}{x}}{1} = 0$ $\boxed{L = 1}$

   c. $\lim_{x \to 2} \frac{x^2 - 4x + 4}{\sin^2 \pi x}$

   $= \lim_{x \to 2} \frac{2x - 4}{\underbrace{2\sin \pi x \cos \pi x}_{= \sin 2\pi x} \cdot \pi} = \lim_{x \to 2} \frac{\cancel{2}}{\pi \cdot \cos 2\pi x \cdot \cancel{2}\pi} = \frac{1}{\pi^2}$

2. Use Newton's Method, $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ to find the zero for the function $f(x) = e^x + x - 5$ for the initial condition $x_0 = 2$. Apply the method for 5 steps and compare your result to the answer given by your calculator.

$f'(x) = e^x + 1$

$x_0 = 2$

$x_1 = 1.4768$

$x_2 = 1.3177$

$x_3 = 1.3066$

$x_4 = 1.306558$

$x_5 = 1.306558641$

Calculator: $x = 1.3065586$ Same

3. Find the following anti-derivatives.

a. $\int 3x^5 - 5x^9 dx$

$\frac{3}{6}x^6 - \frac{5}{10}x^{10} + C = \frac{1}{2}x^6 - \frac{1}{2}x^{10} + C$

b. $\int \frac{t+1}{t} dt = \int 1 + \frac{1}{t} dt = t + \ln t + C$

c. $\int \frac{1}{\sqrt{49-x^2}} dx$

$\arcsin\left(\frac{x}{7}\right) + C$