Math 1151, Quiz #12, Fall 2014 Name KEY

Instructions: Show all work. Answers without work may only receive partial credit. If you are asked for an explanation, explain as completely as possible. Use exact answers unless specifically asked to round.

1. Use the right endpoint rule, and $\lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x_i$ to find the area under the curve $f(x) = 8 - 2x^2$ on the interval $[0,4]$.

$8-2x^2 = f(x) \qquad \Delta x = \frac{4}{n} \qquad x_i = \frac{4i}{n}+0 = \frac{4i}{n}$

$\sum_{i=1}^{n} f(x_i)\Delta x = \sum_{i=1}^{n}\left(8-2\left(\frac{4i}{n}\right)^2\right)\frac{4}{n} = \sum_{i=1}^{n}\left(\frac{32}{n} - \frac{32i^2}{n^2}\cdot\frac{4}{n}\right) = \sum_{i=1}^{n}\left(\frac{32}{n} - \frac{128i^2}{n^3}\right)$

$\sum_{i=1}^{n} \frac{32}{n} - \frac{128}{n^3}\sum i^2 = \frac{32}{\cancel{n}}\cdot\cancel{n} - \frac{\overset{64}{\cancel{128}}}{n^{\cancel{3}2}}\left(\frac{\cancel{n}(n+1)(2n+1)}{\underset{3}{\cancel{6}}}\right) = 32 - \frac{64}{3n^2}(2n^2+3n+1)$

$= 32 - \frac{128\cancel{n^2}}{3\cancel{n^2}} - \frac{64\cancel{n}}{n^{\cancel{2}}} - \frac{64}{3n^2} = 32 - \frac{128}{3} - \frac{64}{n} - \frac{64}{3n^2}$

$\lim_{n\to\infty} -\frac{32}{3} - \frac{64}{n} - \frac{64}{3n^2} = -\frac{32}{3}$

Check $\int_0^4 8-2x^2\,dx = 8x - \frac{2}{3}x^3\Big|_0^4 = 32 - \frac{2(64)}{3} - 0 + 0 =$

$32 - \frac{128}{3} = -\frac{32}{3}$ ok

2. Suppose that $\int_0^3 f(x)dx = 4$, $\int_3^6 f(x)dx = -2$, $\int_3^6 g(x)dx = 9$. Use that to find the following:

a. $\int_0^3 5f(x)dx$ $= 5\int_0^3 f(x)dx = 5(4) = 20$

b. $\int_6^3 (f(x) + 2g(x))dx$ $= \int_6^3 f(x) + \int_6^3 2g(x)\,dx = \int_6^3 f(x) + 2\int_6^3 g(x)\,dx$

$= -\int_3^6 f(x)dx - 2\int_3^6 g(x)dx = -(-2) - 2(9) = 2 - 18 = -16$

3. Use geometry to find the value of $\int_{-2}^{3} |x+1|\,dx$. [Hint: start by sketching the graph.]

$$\int_{-2}^{3} |x+1|\,dx = \int_{-2}^{-1} |x+1|\,dx + \int_{-1}^{3} (x+1)\,dx$$

$$= \frac{1}{2}(1)(1) + \frac{1}{2}(4)(4) =$$

$$\frac{1}{2} + 8 = \quad \frac{17}{2}$$