Stat 2470, Exam #3, Fall 2014 Name KEY

Instructions: Show all work. Use exact answers or appropriate rounding conventions. If you use your calculator, you can show work by saying which calculator commands you used.

1. Two different companies have applied to provide cable television service in a certain region. Let p denote the proportion of all potential subscribers who favour the first company over the second. Consider testing $H_0: p = 0.5, H_a: p \neq 0.5$ based on a sample of 25 individuals who favour the first company and x represent the observed value for X.
   a. Which of the following rejection regions is most appropriate and why? (3 points)
   $$R_1: \{x: x \leq 7 \text{ or } x \geq 18\}, R_2: \{x: x \leq 8\}, R_3: \{x: x \geq 17\}$$ $\alpha = .05$

   R_1 $\neq$ is a 2-taild test, so the rejection region should be two

   b. In the context of this problem, describe the Type I and Type II errors. (6 points)

   Type I: the true proportion of subcribers favouring Company #1 really is about 50%, but your random sample accidentally has much more ~

   Type II: the true proportion is not 50%, but w/ such a small sample, you are not able to prove it.

   c. Compute the probability of a Type II error for the selected region when $p = 0.3$. (5 points)

   $$\Phi\left(\frac{.5-.3+1.96\sqrt{.5(1-.5)/25}}{\sqrt{.3(1-.3)/25}}\right) - \Phi\left(\frac{.5-.3-1.96\sqrt{.5(1-.5)/25}}{\sqrt{.3(1-.3)/25}}\right)$$

   $$= \Phi\left(\frac{.2+.196}{.09165}\right) - \Phi\left(\frac{.2-.196}{.09165}\right) = \Phi(.436) - \Phi(4.321)$$

   normalcdf(-E99, .436) - normalcdf(-E99, 4.321) = .482586

   d. Using the selected region, what would you conclude if 6 of the 25 queried favoured company one? (5 points)

   reject Ho : the probability that the true proportion is close to 50% is small

2. An article described an investigation into the coating weights for large pipes resulting from a galvanized coating process. Production standards call for a true average weight of 200 lbs. per pipe. Using a sample of 30 pipes, the mean of the samples is $\bar{x} = 206.73$ with standard deviation of $s = 6.35$. Test the appropriate hypothesis for this problem. Be sure to include correct hypothesis test notations, and the significance level you choose, as well as your final conclusion stated in terms of the context of the problem. (10 points)

$H_0: \mu = 200$

$H_a: \mu \neq 200$

T-Test Stats

$\mu_0 = 200$

$\bar{X} = 206.73$

$S_x = 6.35$

$n = 30$

$\mu_0 \neq \mu$

$\Rightarrow$ $t = 5.804996554$

$p = 2.723 \times 10^{-6}$

this data suggests we should reject Ho at any reasonable level of significance

pipes weigh more than they should

3. A manufacturer of nickel-hydrogen batteries randomly selects 100 nickel plates for test cells, cycles them a specified number of times, and determines that 14 of the plates have blisters. Does this provide compelling evidence for concluding that more than 10% of all plates blister under such circumstances? State and test the appropriate hypothesis test. (10 points)

1 PropZ test

$H_0: \mu = .1$ $H_a: \mu > .10$

$p_0 = .1$

$x = 14$

$n = 100$

prop > p_0

$\Rightarrow$ $z = 1.333$

$p = .0912$

for a standard α of .05, we would fail to reject Ho and conclude There is not enough evidence to conclude The blister rate is higher than 10%

4. The accompanying table gives summary data on cube compressive strength (N/mm^2) for concrete specimens made with a pulverized fuel-ash mix.

Age (days)	Sample Size	Sample Mean	Sample Standard Deviation
7	68	26.99	4.89
28	74	35.76	6.43

a. Calculate and interpret a 99% confidence interval for the difference between true average 7-day strength and true average 28-day strength. (6 points)

2 SampTInt Stats $\alpha = .01$

$\bar{X}_1 = 26.99$ $\bar{X}_2 = 35.76$

$S_x = 4.89$ $S_x = 6.43$

$n_1 = 68$ $n_2 = 74$

C-level : .99 Pooled: No

$(-11.26, -6.277)$

the true mean difference between the samples is between 6 & 11 N/mm^2

b. Conduct an appropriate hypothesis test to determine if the curing process improves the strength of the concrete. State the hypotheses to be tested, the P-value and your conclusion. (7 points)

2 SampTTest Stats

$H_0: \mu_1 - \mu_2 = 0$

$H_a: \mu_1 - \mu_2 < 0$

(or $\mu_2 > \mu_1$)

$t = -9.1916$

$p = 0$ (very, very small) $< .05$

it does indeed improve the strength of the concreto

5. Suppose that COMPASS math placement scores for a sample of 10 incoming students to CSCC are taken. Then the students undergo a two week remediation program in an attempt to improve their initial placement, and then are retested. Use the information below to determine if the remediation program improved students' placement scores by more than 5 points. Use the significance level of 0.05. (12 points)

Student

	1	2	3	4	5	6	7	8	9	10
No remediation	19	21	25	24	11	20	21	26	18	14
With remediation	21	24	28	29	26	21	20	28	24	26

$\mu_2 - \mu_1 = \Delta$

$H_0: \Delta = 5$

$H_a: \Delta > 5$

$t = -.1257$

$P = .54865 > .05$

TTest Data

$\mu_0 = 5$

List: L_1

$\mu > \mu_0$

remediation does seem to improve placement scores but maybe not by as much as 5 points

6. It is thought that the front cover and nature of the first question on mail surveys influence the response rate. An article tested the theory by experimenting with different cover designs. One cover was plain, and one used the image of a skydiver. The researchers speculated that the return rate would be lower for the plain cover. (10 points)

Cover	**Number Sent**	**Number Returned**
Plain	307	156
Skydiver	388	175

Does this data support the hypothesis of the researchers? Test the relevant hypotheses using $\alpha = 0.1$. State the hypotheses, P-value, and conclusion in context of the problem.

$H_0: p_1 = p_2$
$H_a: p_1 < p_2$

2 Prop 2 test

$X_1 = 156 \quad n_1 = 307$
$X_2 = 175 \quad n_2 = 388$

$Z = 1.497$
$P = .9328$

fail to reject Ho.
it does not appear to make a difference

7. Six samples of each of four types of cereal grain grown in a certain region were analyzed to determine thiamin content, resulting in the following data ($\mu g/g$).

Wheat	5.2	4.5	6.0	6.1	6.7	5.8
Barley	6.5	8.0	6.1	7.5	5.9	5.6
Maize	5.8	4.7	6.4	4.9	6.0	5.2
Oats	8.3	6.1	7.8	7.0	5.5	7.2

Does this data suggest that at least two of the grains differ with respect to true average thiamin content? Use a level $\alpha = 0.05$ test based on the P-value. (12 points)

ANOVA(L_1, L_2, L_3, L_4)

$H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$
H_a: atleast one μ_i different

$F = 3.9565$
$P = .02293$ ← this is < .05 so reject Ho.
at least one is different than the others

factor
df = 3
SS = 8.98
MS = 2.9944

error
df = 20
SS = 15.1366
MS = .7568
S_{xp} = .86996

8. Consider the accompanying table with data on plant growth after the application of five different types of growth hormone.

1:	**13**	**17**	**7**	**14**
2:	21	13	20	17
3:	18	15	20	17
4:	7	11	18	10
5:	6	11	15	8

a. Perform an F-test on the data. Clearly state the hypotheses, test statistic, P-value for the test, and the conclusion. (12 points)

ANOVA$(L_1, L_2, L_3, L_4, L_5)$

$H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$

$H_a: \mu_i \neq \mu_j$ for some i, j

$F = 3.48539984$

$p = .03335772 \leftarrow < .05$

at least one mean is different

factor $df = 4$
$SS = 200.3$
$MS = 50.075$

error $df = 15$
$SS = 215.5$
$MS = 14.3667$

$Sxp = 3.79$

reject H_0

b. If the null hypothesis is rejected, perform Tukey's procedure. (12 points)

$L_1: \bar{x}_1 = 12.75$

$L_2: \bar{x}_2 = 17.75$

$L_3: \bar{x}_3 = 17.5$

$L_4: \bar{x}_4 = 11.5$

$L_5: \bar{x}_5 = 10$

$Q_{0.05,4,15} = 4.05$

$Q_{\alpha,m,\nu}\sqrt{MSE/J} = 4.05\sqrt{14.367/4} = 7.6754$

$\bar{x}_5 = 10$ $\quad \bar{x}_4 = 11.5$ $\quad \bar{x}_1 = 12.75$ $\quad \bar{x}_3 = 17.5$ $\quad \bar{x}_2 = 17.75$

$\bar{x}_5$ is different than $\bar{x}_2$
but none are sufficiently different than
$\bar{x}_4, \bar{x}_1$ or $\bar{x}_3$