Stat 2470, Quiz #4, Fall 2014 Name KEY

Instructions: Show all work. Use exact answers or appropriate rounding conventions. If you use your calculator, you can show work by saying which calculator commands you used.

1. Suppose that you have 130 marbles in a bag. The marbles come in two types, we'll call Type A and Type B. Suppose that there are 100 of the Type A marbles, and 30 of the Type B marbles.
   a. What is the probability that you will select a sample of 10 marbles with 7 Type A marbles and 3 Type B marbles?

$N = 130$
$M = 100$
$n = 10$
$x = 7$

$$\frac{\binom{100}{7}\binom{30}{3}}{\binom{130}{10}} = .2439579\ldots$$

   b. The probability will be slightly different if there are only 98 Type A marbles and 32 Type B marbles. What is the probability of the same event under these circumstances, and does it change the probability in any of the first 4 digits? If so, by how much?

$N = 130$
$M = 98$
$n = 10$
$x = 7$

$$\frac{\binom{98}{7}\binom{32}{3}}{\binom{130}{10}} = .257576\ldots$$

$\Delta p \approx .0136$

2. Suppose that the chances of having a 51% chance of having a girl in any live birth, and a couple decides to keep having kids until they have their first girl. What is the probability that they will need to have more than three kids?

$$1 - (nb(0;.51,1) + nb(1;.51,1) + nb(2;.51,1))$$

$$1 - \left[\binom{0}{0}(.51)^1(.49)^0 + \binom{1}{0}(.51)^1(.49)^1 + \binom{2}{0}(.51)^1(.49)^2\right]$$

$r = 1$
$x = 0,1,2$
$p = .51$

$$= 1 - .778311 = .221689$$

3. The number of people arriving for treatment at an emergency room can be modeled by a Poisson process with a rate parameter of five per hour.
   a. What is the probability that exactly 4 arrivals occur during a particular hour?

$\mu = 5$ $\quad p(4;5) = \text{poissonpdf}(5,4) = .175467\ldots$

   b. What is the probability that at least 4 people arrive during a particular hour?

$$1 - [p(0;5) + p(1;5) + p(2;5) + p(3;5)] =$$
$$1 - \text{poissoncdf}(5,3) = .73497$$

4. The error involved in making a particular measurement is a continuous random variables with probability density function $f(x) = \begin{cases} 0.09375(4-x^2), & -2 \le x \le 2 \\ 0, & otherwise \end{cases}$.

a. Compute $P(X > 0)$

$$\int_0^2 .09375(4-x^2)\,dx = .5$$

b. Compute $P(-1 < X < 1)$

$$\int_{-1}^{1} .09375(4-x^2)\,dx = .6875$$

c. Find the expected value of X.

$$E(X) = \int_{-2}^{2} x[.09375(4-x^2)]\,dx =$$

$$\int_{-2}^{2} .09375(4x - x^3)\,dx = 0$$

(this is predictable because the distribution is symmetric around 0.)