Stat 2470, Quiz #5, Fall 2014 Name KEY

Instructions: Show all work. Use exact answers or appropriate rounding conventions. If you use your calculator, you can show work by saying which calculator commands you used.

1. a. Compute the cumulative distribution function of $f(x) = \begin{cases} 2\left(1 - \frac{1}{x^2}\right), & 1 \le x \le 2 \\ 0, & otherwise \end{cases}$.

$2\int_1^x 1 - \frac{1}{t^2}\,dt = 2\left[t + \frac{1}{t}\right|_1^x = 2\left(x + \frac{1}{x}\right) - 2(1+1) =$

$F(x) = 2x + \frac{2}{x} - 4$

b. What is the 50th percentile (median) of X?

$\left(2x + \frac{2}{x} - 4 = .50 = \frac{1}{2}\right) 2x$

$4x^2 + 4 - 8x = x \qquad 4x^2 - 9x + 4 = 0$

$x = \frac{+9 \pm \sqrt{81 - 4(4)(4)}}{8} = \frac{9 \pm \sqrt{17}}{8} \approx 1.64, \cancel{.6096}$ not between 1 & 2

c. Find V(X).

$E(x^2) = 2\int_1^2 x^2 - 1\,dx = 2\left(\frac{x^3}{3} - x\right|_1^2 =$

$2\left(\frac{8}{3} - 2\right) - 2\left(\frac{1}{3} - 1\right) = \frac{16}{3} - 4 - \frac{2}{3} + 2 = \frac{14}{3} - \frac{6}{3} = \frac{8}{3}$

$E(X) = 2\int_1^2 x - \frac{1}{x}\,dx = 2\left[\frac{1}{2}x^2 - \ln x\right|_1^2 = 2\left[2 - \ln 2 - \frac{1}{2} + 0\right] \approx 1.61$

$8/3 - (1.61)^2 = .0626$

2. If Z is normally distributed, find $P(Z < 1.37)$ and $P(-1.75 < Z < 2.5)$.

normalcdf(-E99, 1.37) = .91465...

normalcdf(-1.75, 2.5) = .95373...

3. If X is normally distributed with a mean of 15 and a standard deviation of 1.25, find $P(X < 17.5)$ and $P(14 < X < 18)$.

normalcdf(-E99, 17.5, 15, 1.25) = .9772497...

normalcdf(14, 18, 15, 1.25) = .7799...