Math 2568, Quiz #9, Fall 2014 Name KEY

Instructions: Show all work. Answer each question as completely as possible. Use exact values (yes, that means fractions!).

1. Find the determinant of $\begin{bmatrix} 1 & 1 & 0 & -2 \\ 1 & 5 & 0 & -1 \\ -2 & -2 & 1 & 3 \\ -3 & 4 & 0 & 8 \end{bmatrix}$ by expanding by rows or columns. Verify the result with the row-reducing method.

$$1\begin{vmatrix} 1 & 1 & -2 \\ 1 & 5 & -1 \\ -3 & 4 & 8 \end{vmatrix} = 1\begin{vmatrix} 5 & -1 \\ 4 & 8 \end{vmatrix} - 1\begin{vmatrix} 1 & -2 \\ -3 & 8 \end{vmatrix} - 2\begin{vmatrix} 1 & 5 \\ -3 & 4 \end{vmatrix} =$$

$$(40+4)-(8-3)-2(4+15) = 44-5-38 = 1$$

$-R_1+R_2 \to R_2$ (1) $\quad 2R_1+R_3 \to R_3$ (1) $\quad 3R_1+R_4 \to R_4$ (1)

$$\begin{bmatrix} 1 & 1 & 0 & -2 \\ 0 & 4 & 0 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 7 & 0 & 2 \end{bmatrix} \Rightarrow 1\begin{vmatrix} 4 & 0 & 1 \\ 0 & 1 & -1 \\ 7 & 0 & 2 \end{vmatrix} = 1\begin{vmatrix} 4 & 1 \\ 7 & 2 \end{vmatrix} = 8-7 = 1$$

2. Solving the system $\begin{cases} 4x_1 + 3x_2 = 10 \\ 2x_1 - 5x_2 = 12 \end{cases}$ by Cramer's Rule.

$A = \begin{bmatrix} 4 & 3 \\ 2 & -5 \end{bmatrix}$ $\det A = -20-6 = -26$

$\text{adj}A_1 = \begin{bmatrix} 10 & 3 \\ 12 & -5 \end{bmatrix}$ $\det \text{adj}A_1 = -50-36 = -86$

$\text{adj}A_2 = \begin{bmatrix} 4 & 10 \\ 2 & 12 \end{bmatrix}$ $\det \text{adj}A_2 = 48-20 = 28$

$x_1 = \frac{-86}{-26} = \frac{43}{13}$ $\quad x_2 = \frac{28}{26} = \frac{14}{13}$ verifies w/ rref.