MAT 223, Exam #2, Fall 2015 Name KEY

Instructions: Show all work. If you use your calculator to answer problems, note the steps taken to enter the data in the calculator and commands used as the "show work" component. It may help to copy all input and output screens, and then interpret the results when answering. Answers without any work may only be worth 1 point per question. Round appropriately for each question.

1. List the events in the sample space for each of the following scenarios. (4 points each)
   a. Toss four fair coins

   HHHH, HHHT, HHTH, HTHH, THHH, HHTT, HTHT, HTTH, THTH, TTHH, THHT, HTTT, THTT, TTHT, TTTH, TTTT

   b. An octahedral die (8-sided) tossed with a tetrahedral die (4-sided).

   (1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (3,4), (4,1), (4,2), (4,3), (4,4), (5,1), (5,2), (5,3), (5,4), (6,1), (6,2), (6,3), (6,4), (7,1), (7,2), (7,3), (7,4), (8,1), (8,2), (8,3), (8,4)

2. For each even below, determine if it is an example of a classical probability, an experimental probability or a personal probability. (3 points each)
   a. There has been just one large terrorist event in the U.S. that caused more than 1000 deaths: 9/11/2001. One CIA analyst thinks the chance of another such attach in the next decade is greater than 50%.

   personal

   b. Of all the police shootings of unarmed civilians in the U.S. last year, fewer than 1% were prosecuted and obtained a conviction.

   empirical

   c. A black jack table uses 8 decks of cards at a time to reduce the benefits of card counting. The probability of getting a king as the first dealt card is 1/13.

   classical

3. Answer each of the following questions based on the contingency table shown here. The data is based on a survey asking respondents how important religion was in their lives. (3 points each)

Gender * Religious_Importance Crosstabulation

Count

		Religious_Importance			
		Fairly	Not	Very	Total
Gender	Female	56	32	39	127
	Male	43	31	25	99
	Total	99	63	64	226

a. If a randomly selected survey was selected from this sample, what is the probability that the respondent considered religion "very important"?

$64/226$

b. If a randomly selected survey was selected from this sample, what is the probability that the respondent considered religion something other than "very important"?

$162/226$

c. If a randomly selected survey was selected from this sample, what is the probability that the respondent was both female and considered religion "not important"?

$32/226$

d. If a randomly selected survey was selected from this sample, what is the probability that the respondent was either female or considered religion "not important"?

$$\frac{127}{226} + \frac{63}{226} - \frac{32}{226} = \frac{158}{226}$$

e. If a randomly selected survey was selected from this sample, what is the probability that the respondent was female given that they considered religion "not important"?

$$\frac{32}{63}$$

f. Given two events (a pair) from the table that are mutually exclusive.

answers will vary:
male/female religion important/not important

g. Are the events "being female" and considered religion "not important" independent? Why or why not? Show work to justify your answer.

$P(\text{not important}) = \frac{63}{226} \approx .27876$

$P(\text{not important} \mid \text{female}) = \frac{32}{127} \approx .2519685\ldots$

These are similar but not the same, so they are dependent.

4. What is the probability of selecting a king from a standard deck, followed by a jack? (4 points)

$\frac{4}{52} \cdot \frac{4}{51} = \frac{16}{2652} \approx .006$

5. Calculate the following by hand. (Reduce first, then you may use calculator to do the final multiplication and division. You can check your work with the calculator, but you must show simplification work on paper.) (4 points each)

a. $5!$ $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 20 \cdot 6 = 120$

b. $\frac{6!}{4!}$ $\frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1} = 30$

c. $\frac{8!}{5!(8-5)!}$ $3!$ $\frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 3 \cdot 2 \cdot 1} = 56$

d. $\frac{7!}{0!}$ $\frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{1} = 5040$

e. $\frac{15!}{2!3!3!}$ $\frac{15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6^{2} \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 3 \cdot 2 \cdot 1 \cdot 3 \cdot 2 \cdot 1} = 1.8162 \times 10^{10}$

6. Find the value of the expressions below (calculator is fine). (3 points each)

a. ${}_{5}^{10}C$ 252

b. $5P3$ 60

c. $\binom{9}{4}$ 126

7. A theatre troupe has 18 actors, and a cast for a play of 9 characters is needed. Assuming the roles can be gender-swapped without difficulty, how many different ways can the play be cast? (4 points)

$18P9 = 1.764 \times 10^{10}$

$18C9$ for ensemble cast

8. A club with thirty-five members nominates 8 people to fill 4 officer positions. Assume that the officer positions will be selected later based on the number of votes cast (so that order does not matter now), how many ways can the slate of 8 nominees be chosen? (4 points)

$35C8 = 23,535,820$

9. Suppose that you want a password that contains only the letters A, Q, U, Z, and X (either upper or lower case), the numbers 1, 4, 7 and the special characters ?, \, [,], { and @. How many such passwords are there if the password is 13 characters long? (5 points)

$5+5+3+6 = 19$

$19^{13} = 4.205 \times 10^{16}$

10. How many ways can the phrase "HE FOUGHT THE LAW" be rearranged into a puzzle. (Treat the three spaces like characters.) (5 points)

$$\frac{17!}{3!\,2!\,2!\,3!} = 2.47 \times 10^{12}$$

H E T

11. Suppose that you have 150 songs on your Spotify playlist. How many ways can you choose eight songs to play on a road trip? (4 points)

$150P8 = 2.1197 \times 10^{17}$ (better)

$150C8 =$

12. What is the probability of all 8 of those songs (see problem 10) will be by the Rolling Stones if you have 23 of their songs on your list? (4 points)

$$\frac{23P8}{150P8} = 9.32 \times 10^{-8}$$

13. What is the probability of getting two sets of 3-of-a-kind in a 7-card poker hand drawn from a standard deck? (5 points)

$$\frac{52(3C2)\cdot 48(3C2)\cdot 44}{52C7} = .007388$$

14. 231 raffle tickets are sold. 18 of those tickets were sold to your friends. What is the probability that the top 4 prizes will all go to your friends? (5 points)

$$\frac{18P4}{231P4} = 2.647 \times 10^{-5}$$

15. Suppose your friend challenges you to a game of chance. You will earn \$4 if you roll a 5, 10 or 12 as a sum of the roll of two standard dice. You will lose \$2 to your friend if you roll a 6, 7, or 8. No money is exchanged for any other outcome. Find the expected value to you of the game? Is the game fair? [Hint: build a table.] (8 points)

x	4	−2	0
p(x)	$\frac{8}{36}$	$\frac{16}{36}$	$\frac{12}{36}$

5: (1,4), (4,1), (2,3), (3,2)
10: (6,4), (4,6), (5,5)
12: (6,6)

6: (1,5), (5,1), (2,4), (4,2), (3,3)
7: (1,6), (6,1), (2,5), (5,2) (3,4), (4,3)
8: (2,6) (6,2) (5,3) (3,5) (4,4)

$$4\left(\frac{8}{36}\right) - 2\left(\frac{16}{36}\right) + 0\left(\frac{12}{36}\right) = \frac{0}{36} = 0$$

the game is fair.

16. Consider a tetrahedral die (4-sided), with sides labeled 1-4. You roll 20 such dice.
    a. What is the expected number of 3s that come up? (3 points)

$$E(X) = np = 20 \cdot \frac{1}{4} = 5$$

    b. What is the probability of getting exactly five 3s? (4 points)

$$\binom{20}{5}\left(\frac{1}{4}\right)^5\left(\frac{3}{4}\right)^{15}$$

or binomialpdf (20, .25, 5) = 20.233%

c. What is the probability of getting ten or more 3s? (5 points)

1 - binomialcdf (20, 1/4, 9) =

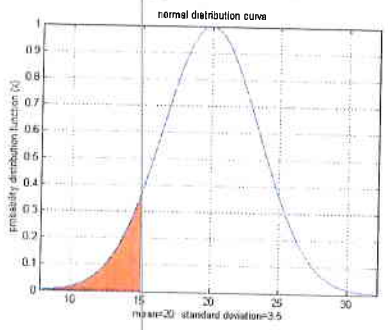
0.01386

17. Suppose that we have a distribution with a mean of 140 with a standard deviation of 12. What is the standard score of the value 122 in this distribution? (4 points)

$$\frac{122-140}{12} = -1.5$$

18. For each of the graphs below, find the probability (area) of the shaded regions. Parts b, c and d are all standard normal distributions with $\mu = 0, \sigma = 1$. For part a, $\mu = 20, \sigma = 3.5$. (5 points each)

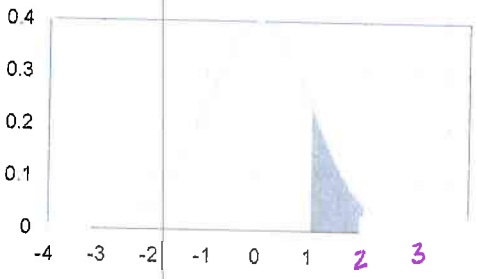
a.



normalcdf (-E99, 15, 20, 3.5)

.07656

b.

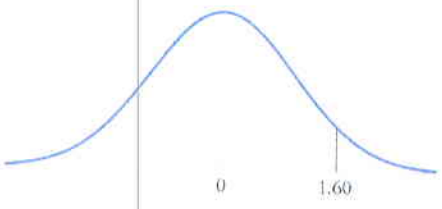


normalcdf (1, 2) =

.1359

13.59%

c.

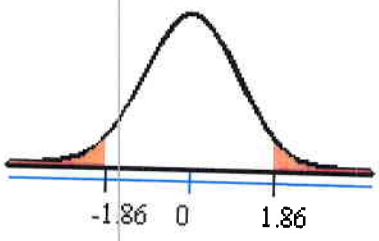


normalcdf (1.6, E99) =

.0548

or 5.48%

d.

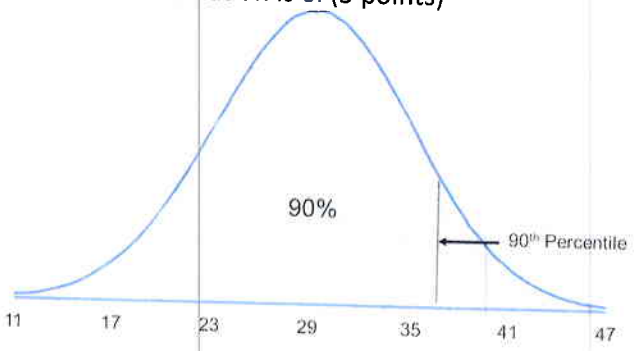


2 x normalcdf(1.86, E99) =

.0628855

or 6.29%

19. On the graph below, find the x-value associated with the 90th percentile if the mean is 29 and the standard deviation is 6. (5 points)



InvNorm(.90 , 29, 6) =

X = 36.69

20. Who has the highest score: a student on a test with a mean of 85 and a standard deviation of 6 scores 90, or a student with a score of 83 with a mean of 72 and a standard deviation of 15? (5 points)

$\frac{90-85}{6} = .8\overline{3}$ ← higher standard score

The first student.

$\frac{83-72}{15} = .7\overline{3}$

or percentiles

normalcdf(-E99, 90, 85, 6) = .79767 ← higher percentile

normalcdf(-E99, 83, 72, 15) = .7683 higher score