MAT 201, Final Exam, Fall 2016 Name KEY

Instructions: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Find the average value of $f(\rho, \theta, \phi) = 5\rho\cos\phi$ over the sphere $x^2 + y^2 + z^2 = 9$. (8 points)

$V = \frac{4}{3}\pi r^3$

$\frac{4}{3}\pi(27)$

$= 36\pi$

$\int_0^{\pi}\int_0^{2\pi}\int_0^{3} 5\rho\cos\varphi\,\rho^2\sin\varphi\,d\rho\,d\theta\,d\varphi = \int_0^{\pi}\int_0^{2\pi}\int_0^{3} 5\rho^3\cos\varphi\sin\varphi\,d\rho\,d\theta\,d\varphi$

$\frac{5}{4}\rho^4\Big|_0^3 \int_0^{\pi}\int_0^{2\pi}\cos\varphi\sin\varphi\,d\theta\,d\varphi = \frac{405}{4}\cdot 2\pi\int_0^{\pi}\cos\varphi\sin\varphi\,d\varphi =$

$\frac{810\pi}{4}\cdot\frac{1}{2}\sin^2\varphi\Big|_0^{\pi} = 0$ $\qquad \frac{0}{36\pi} = 0 = \bar{f}$

2. Find the position vector of a particle with $\vec{a}(t) = 2t\hat{\imath} + 6t^2\hat{\jmath} + 12t^3\hat{k}$, $\vec{v}(0) = \hat{k}$, $\vec{r}(0) = \hat{\jmath}$. (8 points)

$\int 2t\hat{\imath} + 6t^2\hat{\jmath} + 12t^3\hat{k}\,dt = (t^2 + C_1)\hat{\imath} + (2t^3 + C_2)\hat{\jmath} + (3t^4 + C_3)\hat{k}$

$C_1 = 0$ $\quad C_2 = 0$ $\quad C_3 = 1$

$\vec{v}(t) = t^2\hat{\imath} + 2t^3\hat{\jmath} + (3t^4 + 1)\hat{k}$

$\int t^2\hat{\imath} + 2t^3\hat{\jmath} + (3t^4+1)\hat{k}\,dt = (\frac{1}{3}t^3 + C_1)\hat{\imath} + (\frac{1}{2}t^4 + C_2)\hat{\jmath} + (\frac{3}{5}t^5 + t + C_3)\hat{k}$

$C_1 = 0$ $\quad C_2 = 1$ $\quad C_3 = 0$

$\vec{r}(t) = \frac{1}{3}t^3\hat{\imath} + (\frac{1}{2}t^4 + 1)\hat{\jmath} + (\frac{3}{5}t^5 + t)\hat{k}$

3. Use Lagrange multipliers to find the extrema of the function $f(x, y, z) = 2x + 2y + z$ subject to the constraint $x^2 + y^2 + z^2 = 16$. (9 points) $g = x^2 + y^2 + z^2 = 16$

$\nabla f = \langle 2, 2, 1\rangle$

$\nabla g = \langle 2x, 2y, 2z\rangle$

$\nabla f = \lambda\nabla g$

$2 = 2\lambda x \quad \lambda = 1/x$

$2 = 2\lambda y \quad \lambda = 1/y$

$1 = 2\lambda z \quad \lambda = \frac{1}{2z}$

$1/x = 1/y \Rightarrow x = y$

$1/x = 1/2z \Rightarrow x = 2z = y$

$\frac{1}{2}x = z$

$x^2 + x^2 + (\frac{1}{2}x)^2 = 16$

$2x^2 + \frac{1}{4}x^2 = 16$

$\frac{9}{4}x^2 = 16\cdot\frac{4}{9}$

$x^2 = \frac{64}{9}$

$x = \pm\frac{8}{3}$ $\quad y = \pm 8/3$

$z = \pm 4/3$

$(8/3, 8/3, 4/3)$

$(8/3, 8/3, -4/3)$

$(8/3, -8/3, 4/3)$

$(8/3, -8/3, -4/3)$

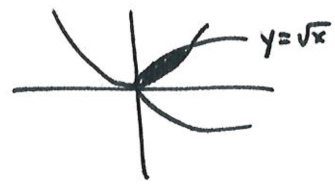
$(-8/3, 8/3, 4/3)$

$(-8/3, 8/3, -4/3)$

$(-8/3, -8/3, 4/3)$

$(-8/3, -8/3, -4/3)$

4. Find the center of mass of the lamina bounded by $y = x^2, x = y^2$ with density $\rho(x, y) = \sqrt{x}$. Set up the three integrals needed. [You do not need to evaluate it.] (8 points)



$$M = \int_0^1 \int_{x^2}^{\sqrt{x}} \sqrt{x}\, dy\, dx$$

$$M_y = \int_0^1 \int_{x^2}^{\sqrt{x}} x^{3/2}\, dy\, dx \qquad \bar{x} = \frac{M_y}{M} \quad \bar{y} = \frac{M_x}{M}$$

$$M_x = \int_0^1 \int_{x^2}^{\sqrt{x}} y\sqrt{x}\, dy\, dx$$

5. A probability density function $f(x, y) = cx^3y$ for $x \geq 0, y \geq 0, y \leq 2 - x$. Find c, and then use it to calculate $P(X \leq 1, Y \leq 1)$. (10 points)

$$\int_0^2 \int_0^{2-x} cx^3y\, dy\, dx = \int_0^2 \frac{c}{2} x^3 y^2 \Big|_0^{2-x} dx =$$

$$\frac{c}{2} \int_0^2 x^3(2-x)^2 dx = \frac{c}{2} \int_0^2 x^3(4-4x+x^2)\, dx =$$

$$\frac{c}{2} \int_0^2 4x^3 - 4x^4 + x^5\, dx = \frac{c}{2}\left[x^4 - \frac{4}{5}x^5 + \frac{1}{6}x^6 \Big|_0^2 \right. =$$

$$\frac{c}{2}\left(16 - \frac{4}{5}(32) + \frac{1}{6}(64)\right) = \frac{8}{15}c = 1 \Rightarrow c = \frac{15}{8}$$

$$P(x \leq 1, y \leq 1) = \int_0^1 \int_0^1 \frac{15}{8} x^3 y\, dy\, dx = \frac{15}{8} \int_0^1 \frac{1}{2} x^3 y^2 \Big|_0^1 dx = \frac{15}{16} \cdot \frac{1}{4} x^4 \Big|_0^1 = \frac{15}{64}$$

6. Set up the four integrals needed to find the center of mass of the volume bounded by the cone $\phi = \frac{\pi}{3}$ and below the sphere $\rho = 4\cos\phi$, with density $\rho(\rho, \theta, \phi) = \rho\theta$. [You do not need to evaluate them.] (10 points)

$$M = \int_0^{\pi/3}\int_0^{2\pi}\int_0^{4\cos\varphi} \rho\theta \cdot \rho^2 \sin\varphi \, d\rho\, d\theta\, d\varphi = \int_0^{\pi/3}\int_0^{2\pi}\int_0^{4\cos\varphi} \rho^3\theta \sin\varphi \, d\rho\, d\theta\, d\varphi$$

$$M_{yz} = \int_0^{\pi/3}\int_0^{2\pi}\int_0^{4\cos\varphi} \underbrace{\rho\cos\theta\sin\varphi}_{x} \cdot \rho^3\theta\sin\varphi \, d\rho\, d\theta\, d\varphi = \int_0^{\pi/3}\int_0^{2\pi}\int_0^{4\cos\varphi} \rho^4\theta\cos\theta\sin^2\varphi \, d\rho\, d\theta\, d\varphi$$

$$M_{xz} = \int_0^{\pi/3}\int_0^{2\pi}\int_0^{4\cos\varphi} \underbrace{\rho\sin\theta\sin\varphi}_{y} \cdot \rho^3\theta\sin\varphi \, d\rho\, d\theta\, d\varphi = \int_0^{\pi/3}\int_0^{2\pi}\int_0^{4\cos\varphi} \rho^4\theta\sin\theta\sin^2\varphi \, d\rho\, d\theta\, d\varphi$$

$$M_{xy} = \int_0^{\pi/3}\int_0^{2\pi}\int_0^{4\cos\varphi} \underbrace{\rho\cos\varphi}_{z} \cdot \rho^3\theta\sin\varphi \, d\rho\, d\theta\, d\varphi = \int_0^{\pi/3}\int_0^{2\pi}\int_0^{4\cos\varphi} \rho^4\theta\cos\varphi\sin\varphi \, d\rho\, d\theta\, d\varphi$$

7. Find the mass of the wire with density $\rho(x, y) = x^2$ over the curve $\vec{r}(t) = 2\sin t\,\hat{\imath} + t\hat{\jmath} - 2\cos t\,\hat{k}$, $[0, \pi]$. (8 points)

$$\int_0^{\pi} \rho\, ds = \int_0^{\pi} 4\sin^2 t \cdot \sqrt{5}\, dt$$

$$2\sqrt{5}\int_0^{\pi} 1 - \cos 2t\, dt =$$

$$2\sqrt{5}\left[t - \tfrac{1}{2}\sin 2t\right|_0^{\pi} = \boxed{2\sqrt{5}\pi}$$

$r'(t) = 2\cos t\,\hat{\imath} + 1\hat{\jmath} + 2\sin t\,\hat{k}$

$\|r'(t)\| = \sqrt{4\cos^2 t + 1 + 4\sin^2 t} = \sqrt{5}$

8. Find the work done through the field $\vec{F}(x,y,z) = (x+y)\hat{\imath} + (y-z)\hat{\jmath} + z^2\hat{k}$ over the path $\vec{r}(t) = t^2\hat{\imath} + t^3\hat{\jmath} + t^2\hat{k}, [0,1]$. (8 points)

$dr = 2t\hat{\imath} + 3t^2\hat{\jmath} + 2t\hat{k}$

$F(t) = (t^2+t^3)\hat{\imath} + (t^3-t^2)\hat{\jmath} + t^4\hat{k}$

$\vec{F}\cdot d\vec{r} = 2t^3 + 2t^4 + 3t^5 - 3t^4 + 2t^5 = 2t^3 - t^4 + 5t^5$

$\int_0^1 2t^3 - t^4 + 5t^5\,dt = \left[\frac{1}{2}t^4 - \frac{1}{5}t^5 + \frac{5}{6}t^6\right|_0^1 =$

$\frac{1}{2} - \frac{1}{5} + \frac{5}{6} = \frac{17}{15}$

9. Evaluate the integral $\int_{-\infty}^{\infty} e^{-\frac{x^2}{2}}\,dx$. [Hint: Use polar coordinates.] (7 points)

$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-\frac{x^2}{2}}\cdot e^{-\frac{y^2}{2}}\,dx\,dy = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{\frac{-x^2-y^2}{2}}\,dA = \int_0^{2\pi}\int_0^{\infty} e^{-r^2/2}\,r\,dr\,d\theta$

$= \int_0^{2\pi} -e^{-r^2/2}\Big|_0^{\infty}\,d\theta = \int_0^{2\pi} 0+1\,d\theta = 2\pi$ (this is square of original integral)

$\int_{-\infty}^{\infty} e^{-x^2/2}\,dx = \sqrt{2\pi}$

10. Find the equation of the tangent plane to the surface $\vec{r}(u,v) = u\sin 2v\,\hat{\imath} + u^2\hat{\jmath} + u\cos 2v\,\hat{k}$ at (4,16,0). (7 points)

$u=4,\ v=\pi/4$

$\vec{r}_u = \sin 2v\,\hat{\imath} + 2u\hat{\jmath} + \cos 2v\,\hat{k}$

$\vec{r}_v = 2u\cos 2v\,\hat{\imath} + 0\hat{\jmath} - 2u\sin 2v\,\hat{k}$

$\vec{r}_u \times \vec{r}_v = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \sin 2v & 2u & \cos 2v \\ 2u\cos 2v & 0 & -2u\sin 2v \end{vmatrix} = (-4u^2\sin 2v - 0)\hat{\imath} - (-2u\sin^2 2v - 2u\cos^2 2v)\hat{\jmath} + (0 - 4u^2\cos 2v)\hat{k}$

$-4u^2\sin^2 v\,\hat{\imath} - 2u\hat{\jmath} - 4u^2\cos 2v\,\hat{k}$

$\langle -64, -8, 0\rangle \Rightarrow \langle 8, 1, 0\rangle$

$8(x-4) + (y-16) = 0$

11. Find the curvature of the function $\vec{r}(t) = t\ln t\,\hat{\imath} + t\hat{\jmath} + e^{-t}\hat{k}$. Evaluate it at $t = 1$. What is the radius of curvature at the same point? (8 points)

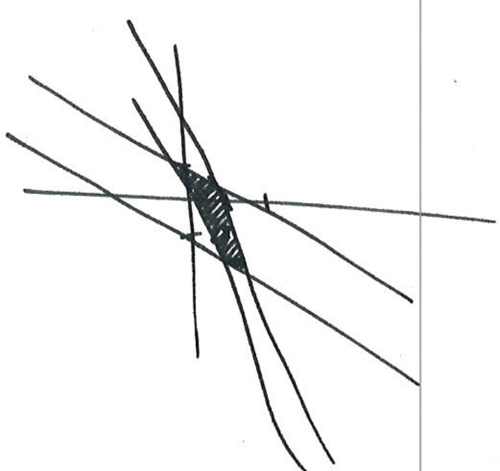
$$r'(t) = (\ln t + 1)\hat{\imath} + 1\hat{\jmath} + (-e^{-t})\hat{k}$$
$$r''(t) = (1/t)\hat{\imath} + 0\hat{\jmath} + e^{-t}\hat{k}$$
$$\vec{r}' \times r''(t) = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \ln t + 1 & 1 & -e^{-t} \\ 1/t & 0 & e^{-t} \end{vmatrix} = (e^{-t} - 0)\hat{\imath} - (e^{-t}\ln t + e^{-t})\hat{\jmath} - \frac{1}{t}\hat{k}$$

$$\|r' \times r''\| = \sqrt{e^{-2t} + (e^{-t}\ln t + e^{-t})^2 + \frac{1}{t^2}} \quad t=1 \Rightarrow \sqrt{e^{-2} + (e^{-1})^2 + 1/1} = \sqrt{2e^{-2}+1}$$

$$\|r'(t)\| = \sqrt{(\ln t + 1)^2 + 1^2 + e^{-2t}} \quad \Rightarrow \sqrt{1^2 + 1^2 + e^{-2}} = \sqrt{2 + e^{-2}}$$

$$K(1) = \frac{\sqrt{2e^{-2}+1}}{(2+e^{-2})^{3/2}} \qquad R(1) = \frac{(2+e^{-2})^{3/2}}{\sqrt{2e^{-2}+1}}$$

12. Sketch the region bounded by $y = 2x - 1, y = 2x + 1, y = 1 - x, y = 3 - x$. Set up a change of variables for the region. Solve for x and y in terms of u and v, and sketch the region after the transformation. (8 points)

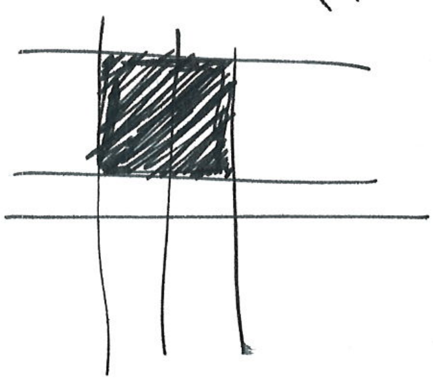


$y - 2x = -1$
$y - 2x = 1$
$y + x = 1$
$y + x = 3$

$u = y - 2x \quad [-1, 1]$
$v = y + x \quad [1, 3]$

$u - v = -3x \Rightarrow x = -\frac{1}{3}(u - v) = \frac{1}{3}(v - u)$

$v = y + \frac{1}{3}(v - u)$
$v = y + \frac{1}{3}v - \frac{1}{3}u$
$\frac{1}{3}u + \frac{2}{3}v = y \quad \Rightarrow y = \frac{1}{3}(u + 2v)$



$x = \frac{1}{3}(v - u)$
$y = \frac{1}{3}(u + 2v)$

13. Consider the function $f(x,y) = \frac{1}{3}y^3 - xy + 4y - \frac{1}{2}x^2 + 6x - 11$. Find $\vec{\nabla} f$. Sketch the graph of the gradient field by graphing the curves $f_x = 0$ and $f_y = 0$. Find any critical points and use the direction field to determine if each critical point is a maximum, a minimum or a saddle point (or cannot be determined). Verify your results with the second partials test. (14 points)

$\langle -y-x+6,\ y^2-x+4 \rangle$

$y = -x+6 \qquad y^2 = x-4$

$(-x+6)^2 = x-4$

$x^2 - 12x + 36 = x - 4$

$x^2 - 13x + 40 = 0$

$(x-8)(x-5) = 0$

$x = 5, 8$

$y = -8+6 = -2$

$y = -5+6 = 1$

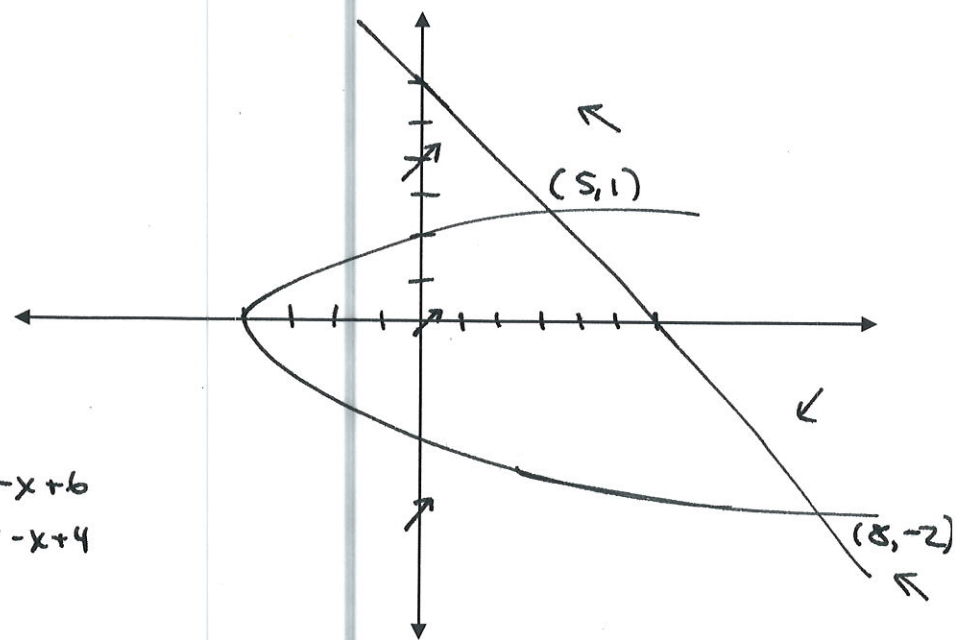
$\nabla f(0,0) = \langle 6,4 \rangle$

$\nabla f(5,3) = \langle -2,9 \rangle$

$\nabla f(0,4) = \langle 2,20 \rangle$

$\nabla f(8,0) = \langle -2,-4 \rangle$

$\nabla f(10,-3) = \langle -1,3 \rangle$



$f_x = -y - x + 6$

$f_y = y^2 - x + 4$

$f_{xx} = -1$

$f_{yy} = 2y$

$f_{xy} = 1$

$D(5,1) = (-1)(2) - (-1)^2 = -3$ saddle point

$D(8,-2) = (-1)(-4) - (-1)^2 = 3$ w/ $f_{xx}, f_{yy} < 0$ maximum

14. Set up an integral to find the surface area of the function $\vec{r}(u,v) = u^2 \cos v\,\hat{\imath} + u^2 \sin v\,\hat{\jmath} + v\hat{k}$ for $0 \le u \le 1, 0 \le v \le \pi$. [You do not need to evaluate it.] (8 points)

$\vec{r}_u = 2u\cos v\,\hat{\imath} + 2u\sin v\,\hat{\jmath} + 0\hat{k}$

$\vec{r}_v = -u^2\sin v\,\hat{\imath} + u^2\cos v\,\hat{\jmath} + 1\hat{k}$

$\vec{r}_u \times \vec{r}_v = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2u\cos v & 2u\sin v & 0 \\ -u^2\sin v & u^2\cos v & 1 \end{vmatrix} =$

$(2u\sin v)\hat{\imath} - (2u\cos v)\hat{\jmath} + (2u^3\cos^2 v + 2u^3\sin^2 v)\hat{k}$

$= (2u\sin v)\hat{\imath} - (2u\cos v)\hat{\jmath} + 2u^3\hat{k}$

$\|\vec{r}_u \times \vec{r}_v\| = \sqrt{4u^2\sin^2 v + 4u^2\cos^2 v + 4u^6}$

$= \sqrt{4u^2 + 4u^6} = 2u\sqrt{1+u^4}$

$\int_0^{\pi}\int_0^1 2u\sqrt{1+u^4}\,du\,dv$

15. Use the Divergence Theorem to calculate the flux through the field $\vec{F}(x, y, z) = x^4\hat{\imath} - x^3z^2\hat{\jmath} + 4xy^2z\hat{k}$ for S: the surface bounded by the cylinder $x^2 + y^2 = 1$ and the planes $z = x + 2$ and $z = 0$. [You do not need to evaluate it.] (8 points)

$$\vec{\nabla} \cdot \vec{F} = 4x^3 + 0 + 4xy^2 = 4x^3 + 4xy^2 = 4x(x^2+y^2)$$
$$= 4r\cos\theta \cdot r^2$$
$$= 4r^3\cos\theta$$

$$\int_0^{2\pi}\int_0^1\int_0^{r\cos\theta+2} 4r^4\cos\theta \, dz\, dr\, d\theta$$

16. Find the a) velocity, b) acceleration and c) speed of a particle defined by the position vector $\vec{r}(t) = \tan t\,\hat{\imath} + \sec t\,\hat{\jmath} + \ln(1+5t)\hat{k}$. (8 points)

$$r'(t) = \sec^2 t\,\hat{\imath} + \sec t \tan t\,\hat{\jmath} + \frac{5}{1+5t}\hat{k}$$

$$r''(t) = 2\sec^2 t\tan t\,\hat{\imath} + (\sec t\tan^2 t + \sec^3 t)\hat{\jmath} + \frac{-25}{(1+5t)^2}\hat{k}$$

$$\|r'(t)\| = \sqrt{\sec^4 t + \sec^2 t\tan^2 t + \frac{25}{(1+5t)^2}}$$

17. Verify that $u(x, y) = \sin x \cosh y + \cos x \sinh y$ satisfies the equation $u_{xx} + u_{yy} = 0$. (5 points)

$u_x = \cos x\cosh y - \sin x \sinh y$

$u_{xx} = -\sin x\cosh y - \cos x \sinh y$

$u_y = \sin x\sinh y + \cos x\cosh y$

$u_{yy}\ \sin x\cosh y + \cos x\sinh y$

$u_{xx} + u_{yy} = -\sin x\cosh y - \cos x\sinh y + \sin x\cosh y + \cos x\sinh y = 0$

18. Set up a double or triple integral to find the volume bounded by $z = 2x^2 + y^2, z = 8 - x^2 - 2y^2$ inside the cylinder $x^2 + y^2 = 1$. [You do not need to evaluate the integral, but it should be as easy to evaluate as possible.] (8 points)

$z_1 = x^2 + x^2 + y^2 = r^2\cos^2\theta + r^2$

$z_2 = 8 - x^2 - y^2 - y^2 = 8 - r^2 - r^2\sin^2\theta$

$z_2 - z_1 = 8 - 2r^2 - r^2\sin^2\theta - r^2\cos^2\theta \quad = 8 - 3r^2$

$\int_0^{2\pi}\int_0^1\int_{r^2\cos^2\theta + r^2}^{8-r^2-r^2\sin^2\theta} r\,dz\,dr\,d\theta$ or $\int_0^{2\pi}\int_0^1 (8-3r^2)r\,dr\,d\theta$

19. Evaluate $\int_0^{\sqrt{\pi}}\int_0^{x}\int_0^{xz} x^2 \sin y \, dydzdx$. (8 points)

$\int_0^{\sqrt{\pi}}\int_0^{x} x^2(-\cos y)\Big|_0^{xz} dz\,dx = -\int_0^{\sqrt{\pi}}\int_0^{x} x^2(\cos(xz) - 1)\,dz\,dx$

$= \int_0^{\sqrt{\pi}} -\frac{x^2\sin(xz)}{x} + z\Big|_0^{x} dx = \int_0^{\sqrt{\pi}} x - x\sin x^2\,dx =$

$\frac{1}{2}x^2 - \frac{1}{2}\cos(x^2)\Big|_0^{\sqrt{\pi}} = \frac{1}{\pi} + (-1/2) - 0 - \frac{1}{2}(1) = \boxed{\frac{\pi}{2} - 1}$

20. Find the potential function, if it exists, for the vector field $\vec{F}(x,y) = 6y^{3/2}\hat{\imath} + 9x\sqrt{y}\hat{\jmath}$. If it does not exist, explain why not. (5 points)

$\vec{\nabla}\times\vec{F} = 3\sqrt{y} - 3\sqrt{y} = 0$ ✓

$\int 2y^{3/2}dx = 2xy^{3/2} + g(y)$

$\int 3xy^{1/2}dy = 3x\cdot\frac{2}{3}y^{3/2}, h(x) \quad 2xy^{3/2} + h(x)$

$f(x,y) = 2xy^{3/2} + K$

21. Find the volume of the parallelepiped determined by the vectors $\langle 4,1,2\rangle, \langle 3,3,-1\rangle, \langle 5,8,1\rangle$. (5 points)

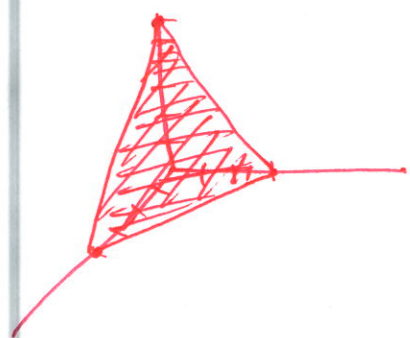
$$\begin{vmatrix} 4 & 1 & 2 \\ 3 & 3 & -1 \\ 5 & 8 & 1 \end{vmatrix} =$$

$4(3+8) - 1(3+5) + 2(24-15) =$

$4(11) - 1(8) + 2(9) = 44 - 8 + 18 = 54$

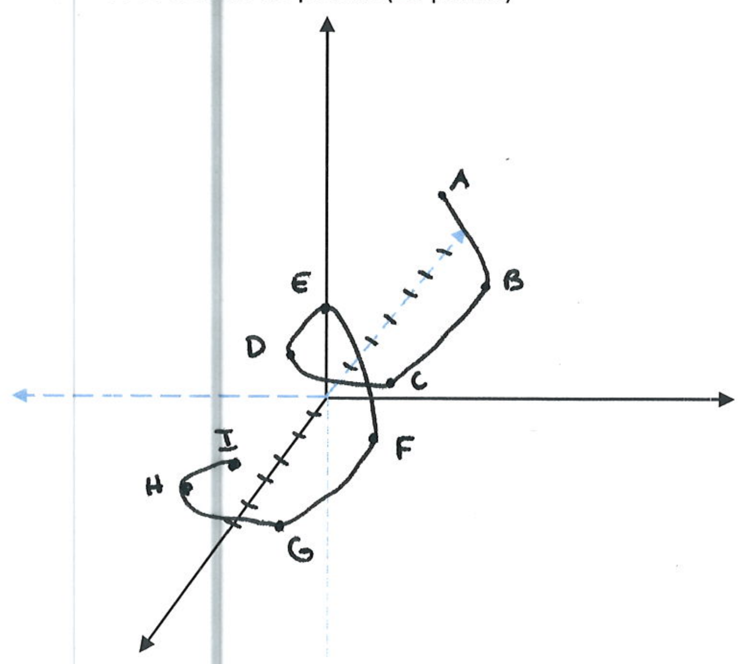
22. Use intercepts to graph the plane $3x + 4y + 2z = 12$. (5 points)

$(4,00)$
$(0,3,0)$
$(0,0,6)$



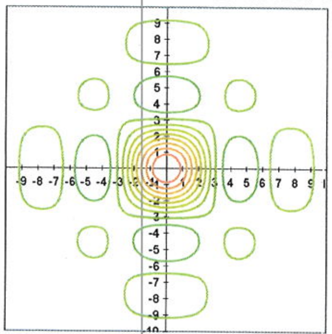
23. Sketch the graph of $\vec{r}(t) = t\hat{\imath} + \sin t\,\hat{\jmath} + \cos t\,\hat{k}$. Plot at around 10 points. (13 points)

t	x	y	z	
-2π	-2π	0	1	A
$-3\pi/2$	$-3\pi/2$	1	0	B
$-\pi$	$-\pi$	0	-1	C
$-\pi/2$	$-\pi/2$	-1	0	D
0	0	0	1	E
$\pi/2$	$\pi/2$	1	0	F
π	π	0	-1	G
$3\pi/2$	$3\pi/2$	-1	0	H
2π	2π	0	1	I



24. Match each function to its set of level curves. (3 points each)

v

a.


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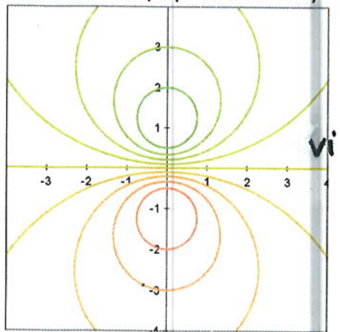
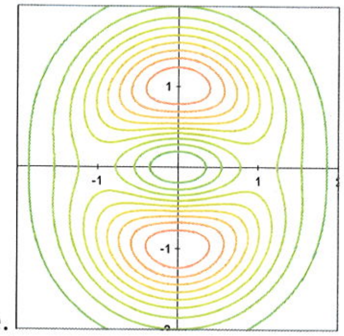
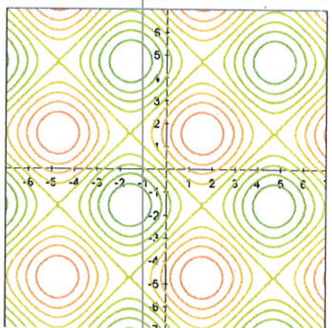


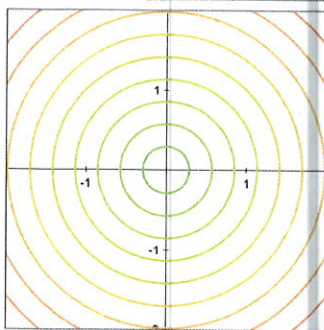
vi

e.


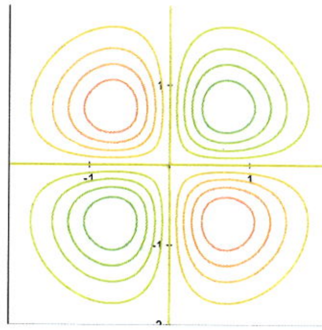
i

iv

b.


d.


ii

f.


iii

i. $z = (x^2+3y^2)e^{-x^2-y^2}$ E

ii. $z = \sqrt{x^2+y^2}$ D

iii. $z = xye^{-x^2-y^2}$ F

iv. $z = \sin x + \sin y$ B

v. $z = \frac{\sin x \sin y}{xy}$ A

vi. $z = -\frac{3y}{x^2+y^2+1}$ C

25. Find the limit. (7 points each)

a. $\lim_{(x,y,z)\to(0,0,0)} \frac{xz+yz}{x^2+y^2+z^2}$

$$\lim_{\rho\to 0} \frac{\rho^2 \sin^2\varphi\cos\theta\sin\theta + \rho^2\sin\varphi\sin\theta\cos\varphi}{\rho^2}$$

$$\lim_{\rho\to 0} \sin^2\varphi\cos\theta\sin\theta + \sin\varphi\sin\theta\cos\varphi$$

DNE

since value depends on path

b. $\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2+y^4}$

$x = ky^4$

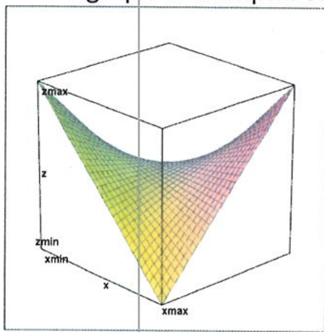
$$\lim_{y\to 0} \frac{ky^4y^4}{k^2y^8+y^8} = \lim_{y\to 0}\frac{k\cancel{y^8}}{\cancel{y^8}(k^2+1)}$$

$$= \frac{k}{k^2+1}$$

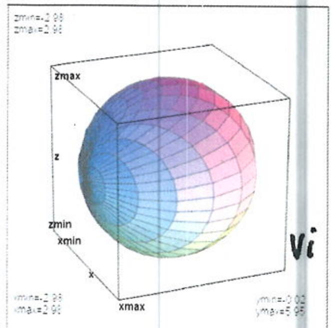
DNE

since value depends on path

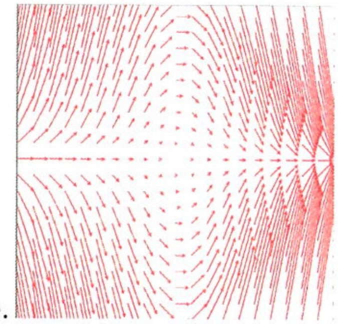
26. Match the graph to its equation. (3 point each)

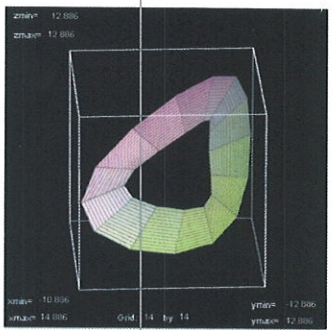
iv


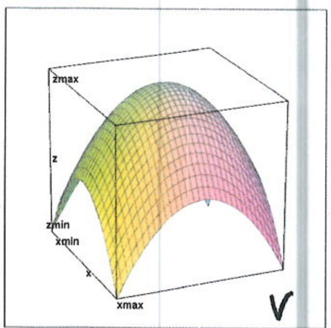
a.



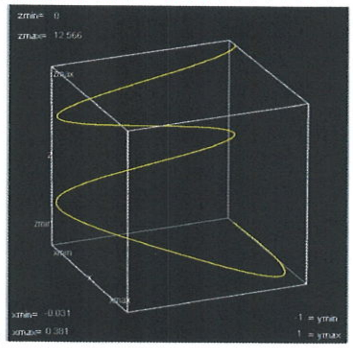
c. vi


e. i

iii

b.



d. V


f. ii

i. $\vec{F}(x,y) = \sqrt{x^2+y^2}\,\hat{\imath} - xy\,\hat{\jmath}$ E iv. $z = xy$ A

ii. $\vec{r}(t) = e^{-0.8t}\sin t\,\hat{\imath} + \cos t\,\hat{\jmath} + t\hat{k}$ F v. $z = 4 - r^2$ D vi. $\rho = 6\sin\theta\sin\phi$ C

iii. $\vec{r}(u,v) = \left(2\cos u + v\cos\left(\frac{u}{2}\right)\right)\hat{\imath} + \left(2\sin u + v\cos\left(\frac{u}{2}\right)\right)\hat{\jmath} + v\sin\left(\frac{u}{2}\right)\hat{k}$ B

27. Describe the difference between a sink, a source and an incompressible fluid in the context of the Divergence Theorem. (6 points)

a sink has a flow which is negative through a closed surface. (more goes in than comes out)

a source has a flow which is positive through a closed surface (more comes out than goes in)

an incompressible fluid has a flow which is zero through a closed surface
(the same goes in as comes out)