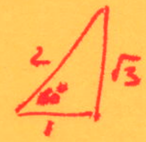
201 Homework #1 Key

1a. $\vec{v} = 5\cos(120°)\,\hat{\imath} + 5\sin(120°)\,\hat{\jmath}$

$= 5\left(-\tfrac{1}{2}\right)\hat{\imath} + 5\left(\tfrac{\sqrt{3}}{2}\right)\hat{\jmath} = -\frac{5}{2}\hat{\imath} + \frac{5\sqrt{3}}{2}\hat{\jmath}$



b. $\vec{v} = 8\cos(-3.5)\,\hat{\imath} + 8\sin(-3.5)\,\hat{\jmath}$

$\approx -7.49\,\hat{\imath} + 2.81\,\hat{\jmath}$

2. $r = \sqrt{20^2 + 20^2} = \sqrt{800} = 20\sqrt{2} = \|\vec{v}\|$

$\theta = \tan^{-1}\left(\frac{20}{20}\right) = \pi/4$

3. a. i. $\vec{AB} = \langle 7-5,\ 1-3,\ 3-4\rangle = \langle 2, -2, -1\rangle$

$\vec{AC} = \langle 3-5,\ 5-3,\ 3-4\rangle = \langle -2, 2, -1\rangle$

$\vec{BC} = \langle 3-7,\ 5-1,\ 3-3\rangle = \langle -4, 4, 0\rangle$

ii. $\|\vec{AB}\| = \sqrt{4+4+1} = \sqrt{9} = 3$

$\|\vec{AC}\| = \sqrt{4+4+1} = \sqrt{9} = 3$

$\|\vec{BC}\| = \sqrt{16+16+0} = \sqrt{32} = 4\sqrt{2}$

iii. isosceles since two sides are the same
(not right since $9+9 \neq 32$)

iv. $\vec{BC}$ is long side $\left(\frac{10}{2}, \frac{6}{2}, \frac{6}{2}\right) = (5, 3, 3)$

v. $\cos\theta = \dfrac{-4 + (-4) + 1}{3 \cdot 3} = \dfrac{-7}{9} \Rightarrow 2.46$ radians $> \pi/2$ obtuse
(There can only be one obtuse angle so we can stop here.)

vi. $A = \frac{1}{2}\left\| \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & -2 & -1 \\ -2 & 2 & -1 \end{vmatrix} \right\| = \frac{1}{2}\left| \underbrace{(2+2)}_{4}\hat{\imath} - \underbrace{(-2-2)}_{4}\hat{\jmath} + \underbrace{(4-4)}_{0}\hat{k} \right| = \frac{1}{2}\sqrt{16+16+0} =$

$\frac{1}{2}(4\sqrt{2}) = 2\sqrt{2}$

b. i. $\vec{AB} = \langle 0-5,\ 2-0,\ 0-0\rangle = \langle -5, 2, 0\rangle$

$\vec{AC} = \langle 0-5,\ 0-0,\ -3-0\rangle = \langle -5, 0, -3\rangle$

$\vec{BC} = \langle 0-0,\ 0-2,\ -3-0\rangle = \langle 0, -2, -3\rangle$

201 Homework #1 Key

36 cont'd.

ii. $\|\vec{AB}\| = \sqrt{25+4+0} = \sqrt{29}$

$\|\vec{AC}\| = \sqrt{25+9+0} = \sqrt{34}$

$\|\vec{BC}\| = \sqrt{0+4+9} = \sqrt{13}$

iii. $(\sqrt{29})^2 + (\sqrt{13})^2 = 29+13 = 42 \neq \sqrt{34}$ neither

iv. $\vec{AC}$ is longest side $\left(\frac{5}{2}, 0, -\frac{3}{2}\right)$

v. $\cos\theta = \dfrac{25+0+0}{\sqrt{29}\sqrt{34}} \approx .65$ radians $< \pi/2$ acute $\angle BAC$

$\cos\theta = \dfrac{0+4+0}{\sqrt{29}\sqrt{13}} \approx 1.36$ radians $< \pi/2$ acute $\angle ABC$

$\cos\theta = \dfrac{0+0+9}{\sqrt{34}\sqrt{13}} \approx 1.128$ radians $< \pi/2$ acute $\angle BCA$

acute triangle

iv. $A = \frac{1}{2}\left\| \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -5 & 2 & 0 \\ -5 & 0 & -3 \end{vmatrix} \right\| = \frac{1}{2}\left\| \underbrace{(-6-0)}_{-6}\hat{i} - \underbrace{(15-0)}_{-15}\hat{j} + \underbrace{(0+10)}_{10}\hat{k} \right\| = \frac{1}{2}\sqrt{36+225+100} = \frac{1}{2}\sqrt{361}$

$= \frac{19}{2}$

4.

a. $(x^2+6x+9) + (y^2-2y+1) + (z^2+10z+25) = 19+9+1+25$

$(x+3)^2 + (y-1)^2 + (z+5)^2 = 54$

Center $(-3, 1, -5)$ radius $= \sqrt{54} = 3\sqrt{6}$

b. $2x^2+2y^2+2z^2-8x+24z = 1$ $\div 2$

$(x^2-4x+4) + y^2 + (z^2+12z+36) = 1/2 + 4 + 36$

$(x-2)^2 + y^2 + (z+6)^2 = \frac{81}{2}$

Center $(2, 0, -6)$ radius $\frac{9}{\sqrt{2}}$

c. $(x-2)^2 + (y+4)^2 + (z-1)^2 = 25$

d. $(2,0,0)$, $(0,6,0)$ midpoint = center $r = \sqrt{(1-0)^2+(3-6)^2+(0)^2} = \sqrt{1+9+0} = \sqrt{10}$

$\left(\frac{2}{2}, \frac{6}{2}, 0\right) = (1,3,0)$ $(x-1)^2 + (y-3)^2 + z^2 = 10$

201 Homework #1 Key

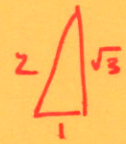
5. $\cos\alpha = \dfrac{-4+0+0}{\sqrt{1}\,\sqrt{16+9+25}} = \dfrac{-4}{\sqrt{50}} = \dfrac{-4}{5\sqrt{2}}$ $\alpha \approx .844$ radians

$\cos\beta = \dfrac{3}{\sqrt{50}}$ $\beta \approx 1.13$ radians

$\cos\gamma = \dfrac{5}{\sqrt{50}}$ $\gamma \approx \pi/4$

6. Since $\vec{u}\cdot\vec{v} = \|\vec{u}\|\|\vec{v}\|\cos\theta$ and since $|\cos\theta| < 1$

$\Rightarrow |\vec{u}\cdot\vec{v}| \le \|\vec{u}\|\|\vec{v}\|$.



7. $\vec{F}_1 = 20\cos 45^\circ\,\hat{\imath} + 20\sin 45^\circ\,\hat{\jmath} = \dfrac{20}{\sqrt{2}}\hat{\imath} + \dfrac{20}{\sqrt{2}}\hat{\jmath}$

$\vec{F}_2 = 5\cos(-60^\circ)\,\hat{\imath} + 5\sin(-60^\circ)\,\hat{\jmath} = +\dfrac{5}{2}\hat{\imath} - \dfrac{5\sqrt{3}}{2}\hat{\jmath}$

$\vec{F}_3 = 30\cos(-120^\circ)\,\hat{\imath} + 30\sin(-120^\circ)\,\hat{\jmath} = -\dfrac{30}{2}\hat{\imath} - \dfrac{30\sqrt{3}}{2}\hat{\jmath} = -15\hat{\imath} - 15\sqrt{3}\,\hat{\jmath}$

$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 1.64\,\hat{\imath} - 16.17\,\hat{\jmath}$ $\|\vec{F}_1+\vec{F}_2+\vec{F}_3\| = \sqrt{1.64^2 + 16.17^2} = 16.25$

$\tan^{-1}\left(\dfrac{-16.17}{1.64}\right) = -1.4697$ radians or -84.2°

8. $\text{proj}_v\,\vec{u} = \left(\dfrac{3-4-6}{9+16+4}\right)\langle 3,4,-2\rangle = \dfrac{-7}{29}\langle 3,4,-2\rangle = \left\langle \dfrac{-21}{29}, \dfrac{-28}{29}, \dfrac{14}{29}\right\rangle$

9. $\langle a,b,c\rangle \cdot \langle 4,0,2\rangle = 0$

$4a + 2c = 0 \Rightarrow \dfrac{4a}{2} = \dfrac{-2c}{2} \Rightarrow 2a = -c$ $a = 1$ $c = -2$

$\langle 1,0,-2\rangle$ or any multiple of this vector.

(b can be anything $\Rightarrow$ $\langle 0,b,0\rangle$ is also $\perp$

10. $W = \vec{f}\cdot\vec{d}$ $\vec{d} = \langle 6,2,12\rangle$

$\langle 8,-6,9\rangle \cdot \langle 6,2,12\rangle = 48 - 12 + 108 = 144$