MAT 201, Quiz #12, Fall 2016 Name KEY.

Instructions: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Find the position of the particle with $\vec{a}(t) = t\hat{\imath} + e^t\hat{\jmath} + e^{-t}\hat{k}, \vec{v}(0) = \hat{\imath}, \vec{r}(0) = 2\hat{\jmath} + 3\hat{k}$.

$$\int t\hat{\imath} + e^t\hat{\jmath} + e^{-t}\hat{k}\,dt = \left(\tfrac{1}{2}t^2 + c_1\right)\hat{\imath} + (e^t + c_2)\hat{\jmath} + (-e^{-t} + c_3)\hat{k} = \vec{v}(t)$$

$$c_1 = 1 \qquad c_2 = -1 \qquad c_3 = 1$$

$$\vec{v}(t) = \left(\tfrac{1}{2}t^2 + 1\right)\hat{\imath} + (e^t - 1)\hat{\jmath} + (1 - e^{-t})\hat{k}$$

$$\int \left(\tfrac{1}{2}t^2 + 1\right)\hat{\imath} + (e^t - 1)\hat{\jmath} + (1 - e^{-t})\hat{k}\,dt =$$

$$\left(\tfrac{1}{6}t^3 + t + c_1\right)\hat{\imath} + (e^t - t + c_2)\hat{\jmath} + (t + e^{-t} + c_3)\hat{k}$$

$$c_1 = 0 \qquad c_2 = 1 \qquad c_3 = 2$$

$$\vec{r}(t) = \left(\tfrac{1}{6}t^3 + t\right)\hat{\imath} + (e^t - t + 1)\hat{\jmath} + (t + e^{-t} + 2)\hat{k}$$

2. Use Lagrange multipliers to find the extrema of the function $f(x, y, z) = xyz$ subject to $x^2 + y^2 + 3z^2 = 6$.

$g = x^2 + y^2 + 3z^2 - 6$

$\nabla f = \langle yz, xz, xy \rangle$

$\nabla g = \langle 2x, 2y, 6z \rangle$

$yz = 2\lambda x \Rightarrow \lambda = \dfrac{yz}{2x}$

$xz = 2\lambda y \Rightarrow \lambda = \dfrac{xz}{2y}$

$xy = 6\lambda z \Rightarrow \lambda = \dfrac{xy}{6z}$

$\dfrac{yz}{\cancel{2}x} = \dfrac{xz}{\cancel{2}y} \Rightarrow y^2z = x^2z$

$z(x^2 - y^2) = 0$

$z = 0$ or $x^2 = y^2$

$\dfrac{yz}{2x} = \dfrac{xy}{6z} \Rightarrow \cancel{6}_3 yz^2 = 2x^2y$

$y = 0$ or $(3z^2 - x^2) = 0$

$3z^2 = x^2$

$x^2 + y^2 + 3z^2 = 6$

$x^2 + y^2 + x^2 = 6 \qquad 3z^2 = x^2$

$x^2 + x^2 + x^2 = 6 \qquad x^2 = y^2$

$3x^2 = 6$

$x^2 = 2 \qquad x = \pm\sqrt{2}$

$y = \pm\sqrt{2}$

$z = \pm\sqrt{2/3}$

$(0, 0, 0)$

$(\sqrt{2}, \sqrt{2}, \sqrt{2/3}) \qquad (-\sqrt{2}, -\sqrt{2}, \sqrt{2/3})$

$(\sqrt{2}, \sqrt{2}, -\sqrt{2/3}) \qquad (-\sqrt{2}, -\sqrt{2}, -\sqrt{2/3})$

$(\sqrt{2}, -\sqrt{2}, \sqrt{2/3})$

$(\sqrt{2}, -\sqrt{2}, -\sqrt{2/3})$

$(-\sqrt{2}, \sqrt{2}, \sqrt{2/3})$

$(-\sqrt{2}, \sqrt{2}, -\sqrt{2/3})$