MAT 201, Quiz #6, Fall 2016 Name KEY

Instructions: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Find $\vec{\nabla} f$ and $\vec{\nabla}^2 f$ for $f(x, y) = xe^{xy}$.

$$\nabla f = \langle e^{xy} + xye^{xy},\ x^2e^{xy} \rangle$$

$$ye^{xy} + ye^{xy} + xy^2e^{xy} + x^3e^{xy} =$$

$$\nabla^2 f = 2ye^{xy} + xy^2e^{xy} + x^3e^{xy}$$

2. Find $\vec{\nabla} \times \vec{F}$ and $\vec{\nabla} \cdot \vec{F}$ for $\vec{F}(x, y, z) = xy^2z^3\hat{\imath} + x^3yz^2\hat{\jmath} + x^2y^3z\hat{k}$.

$$\vec{\nabla}\times\vec{F} \quad \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ xy^2z^3 & x^3yz^2 & x^2y^3z \end{vmatrix} = (3x^2y^2z - 2x^3yz)\hat{\imath} - (2xy^3z - 3xy^2z^2)\hat{\jmath} + (3x^2yz^2 - 2xyz^3)\hat{k}$$

$$\vec{\nabla}\cdot\vec{F} = y^2z^3 + x^3z^2 + x^2y^3$$

3. Find $\int\int_D xy^2 dA$ where D is the region $x = 0, x = \sqrt{1-y^2}$ [Hint: it may be helpful to use polar.] Sketch a graph of the region.

$$\int_{-\pi/2}^{\pi/2}\int_0^1 r\cos\theta\, r^2\sin^2\theta\, r\,dr\,d\theta$$

$$= \int_{-\pi/2}^{\pi/2}\int_0^1 r^4\cos\theta\sin^2\theta\,dr\,d\theta =$$

$$x = \sqrt{1-y^2} \Rightarrow x^2+y^2 = 1$$

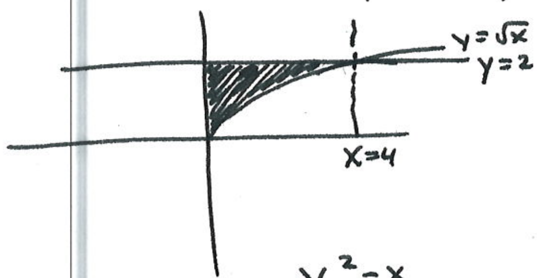
$$\theta \in [-\pi/2, \pi/2]$$

$$\int_{-\pi/2}^{\pi/2} \frac{1}{5}r^5\Big|_0^1 \cos\theta\sin^2\theta\,d\theta = \int_{-\pi/2}^{\pi/2}\frac{1}{5}\sin^2\theta\cos\theta\,d\theta = \frac{1}{5}\cdot\frac{1}{3}\sin^3\theta\Big|_{-\pi/2}^{\pi/2} =$$

$$\frac{1}{15}\left[1-(-1)\right] = \boxed{\frac{2}{15}}$$

4. Find $\int_0^4 \int_{\sqrt{x}}^2 \frac{1}{y^3+1}\, dy dx$ [Hint: switch order of integration.] Sketch a graph of the region.

$y=\sqrt{x} \quad y=2$



$y^2=x$

$$\int_0^2 \int_0^{y^2} \frac{1}{y^3+1}\, dx\, dy$$

$$= \int_0^2 \frac{x}{y^3+1}\Big|_0^{y^2} dy = \int_0^2 \frac{y^2}{y^3+1}\, dy$$

$u = y^3+1$

$du = 3y^2\, dy$

$\frac{1}{3} du = y^2 dy$

$\frac{1}{3}\int \frac{1}{u}\, du$

$$= \frac{1}{3} \ln|y^3+1|\Big|_0^2$$

$$= \frac{1}{3}\left[\ln(9) - \underbrace{\ln(1)}_{0}\right] =$$

$$\boxed{\frac{1}{3}\ln 9}$$