

Instructions: Show all work. Some problems will instruct you to complete operations by hand, some can be done in the calculator. To show work on calculator problems, show the commands you used, and the resulting matrices. **Give exact answers** (yes, that means fractions, square roots and exponentials, and not decimals) unless specifically directed to give a decimal answer. This will require some operations to be done by hand even if not specifically directed to. Be sure to complete all parts of each question.

1. For the matrix $B = \begin{bmatrix} 2 & -1 \\ -1 & 8 \end{bmatrix}$, find the following:

a. B^{-1} (using the formula, not your calculator)

$$\frac{1}{16-1} \begin{bmatrix} 8 & 1 \\ 1 & 2 \end{bmatrix} = \frac{1}{15} \begin{bmatrix} 8 & 1 \\ 1 & 2 \end{bmatrix}$$

b. $B^{-1}B$

$$\frac{1}{15} \begin{bmatrix} 8 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -1 & 8 \end{bmatrix} = \frac{1}{15} \begin{bmatrix} 16-1 & -8+8 \\ 2-2 & -1+16 \end{bmatrix} = \frac{1}{15} \begin{bmatrix} 15 & 0 \\ 0 & 15 \end{bmatrix} \\ = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

2. For the B matrix above, write the elementary matrices that would be needed to reduce B to an upper triangular matrix, and use that to construct the LU factorization of B .

$$\begin{bmatrix} 2 & -1 \\ -1 & 8 \end{bmatrix} \quad R_1 + 2R_2 \rightarrow R_2 \quad \begin{bmatrix} 2 & -1 \\ 0 & 15 \end{bmatrix}$$

$$E_1 = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}$$

$$U = \begin{bmatrix} 2 & -1 \\ 0 & 15 \end{bmatrix}$$

$$L = E_1^{-1} = \begin{bmatrix} 1 & 0 \\ -1/2 & 1/2 \end{bmatrix}$$

(answers may vary)