MAT 202, Quiz #7, Fall 2016 Name KEY

Instructions: Show all work. Some problems will instruct you to complete operations by hand, some can be done in the calculator. To show work on calculator problems, show the commands you used, and the resulting matrices. **Give exact answers** (yes, that means fractions, square roots and exponentials, and not decimals) unless specifically directed to give a decimal answer. This will require some operations to be done by hand even if not specifically directed to. Be sure to complete all parts of each question.

1. For the matrix $A = \begin{bmatrix} 1 & 2 & 1 & 0 & 0 \\ 2 & 5 & 1 & 1 & 0 \\ 3 & 7 & 2 & 2 & -2 \\ 4 & 9 & 3 & -1 & 4 \end{bmatrix}$, find the following:

rref $\Rightarrow \begin{bmatrix} 1 & 0 & 3 & 0 & -4 \\ 0 & 1 & -1 & 0 & 2 \\ 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

$x_1 = -3x_3 + 4x_5$
$x_2 = x_3 - 2x_5$
$x_3 = x_3$
$x_4 = 2x_5$
$x_5 = x_5$

a. A basis for $Col\ A$

$\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \\ 7 \\ 9 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \\ -1 \end{bmatrix} \right\}$

b. A basis for $Nul\ A$

$\left\{ \begin{bmatrix} -3 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ -2 \\ 0 \\ 2 \\ 1 \end{bmatrix} \right\}$

c. $Rank\ A$

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2. A matrix is 7×6.
   a. What is the largest dimension possible of the column space?

6 since there can only be six pivots

   b. What is the smallest possible dimension of the null space?

0 (There could be a pivot in every column meaning no free variables)

3. A basis for R^2 is $B = \left\{ \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \end{bmatrix} \right\}$. Express $\vec{x} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ as a vector in the B-basis, i.e. find $[\vec{x}]_B$.

$P_B = \begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix}$ $\quad P_B [\vec{x}]_B = \vec{x}$

$P_B^{-1} \vec{x} = [\vec{x}]_B$

$\frac{-1}{2}\begin{bmatrix} 5 & -4 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} -1 \\ 1 \end{bmatrix} = -\frac{1}{2}\begin{bmatrix} -5-4 \\ 3+2 \end{bmatrix} = -\frac{1}{2}\begin{bmatrix} -9 \\ 5 \end{bmatrix} = \begin{bmatrix} 9/2 \\ -5/2 \end{bmatrix}$