

# Table of Integrals

## Basic Rules

$$(1.1) \quad \int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$(1.2) \quad \int \frac{1}{x} dx = \ln|x| + C$$

$$(1.3) \quad \int e^x dx = e^x + C$$

$$(1.4) \quad \int a^x dx = \frac{a^x}{\ln a} + C$$

$$(1.5) \quad \int \sin x dx = -\cos x + C$$

$$(1.6) \quad \int \cos x dx = \sin x + C$$

$$(1.7) \quad \int \tan x dx = -\ln|\cos x| + C = \ln|\sec x| + C$$

$$(1.8) \quad \int \cot x dx = \ln|\sin x| + C$$

$$(1.9) \quad \int \sec^2 x dx = \tan x + C$$

$$(1.10) \quad \int \sec x dx = \ln|\sec x + \tan x| + C = \ln\left|\frac{1 + \sin x}{\cos x}\right| + C$$

$$(1.11) \quad \int \csc^2 x dx = -\cot x + C$$

$$(1.12) \quad \int \csc x dx = -\ln|\csc x + \cot x| + C = \ln\left|\frac{\sin x}{1 + \cos x}\right| + C$$

$$(1.13) \quad \int \sec x \tan x dx = \sec x + C$$

$$(1.14) \quad \int \csc x \cot x dx = -\csc x + C$$

$$(1.15) \quad \int \frac{1}{\sqrt{1-x^2}} dx = \arcsin x + C$$

$$(1.16) \quad \int \frac{1}{x\sqrt{x^2-1}} dx = \operatorname{arcsec}|x| + C$$

$$(1.17) \quad \int \frac{1}{1+x^2} dx = \arctan x + C$$

$$(1.18) \quad \int \sinh x dx = \cosh x + C$$

$$(1.19) \quad \int \cosh x dx = \sinh x + C$$

$$(1.20) \quad \int \tanh x dx = \ln|\cosh x| + C$$

$$(1.21) \quad \int \operatorname{sech}^2 x dx = \tanh x + C$$

$$(1.22) \quad \int \coth x dx = \ln |\sinh x| + C$$

$$(1.23) \quad \int \operatorname{sech} x \tanh x dx = -\operatorname{sech} x + C$$

$$(1.24) \quad \int \operatorname{csch} x \coth x dx = -\operatorname{csch} x + C$$

$$(1.25) \quad \int \operatorname{csch}^2 x dx = -\coth x + C$$

$$(1.26) \quad \int \frac{1}{\sqrt{x^2+1}} dx = \sinh^{-1} x + C$$

$$(1.27) \quad \int \frac{1}{\sqrt{x^2-1}} dx = \cosh^{-1} x + C$$

$$(1.28) \quad \int \frac{1}{1-x^2} dx = \tanh^{-1} x + C$$

$$(1.29) \quad \int \frac{1}{x\sqrt{1-x^2}} dx = -\operatorname{sech}^{-1} x + C$$

$$(1.30) \quad \int \frac{1}{x\sqrt{1+x^2}} dx = -\operatorname{csch}^{-1} x + C$$

### Rational and Polynomial Functions

$$(2.1) \quad \int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C, n \neq -1$$

$$(2.2) \quad \int x(ax+b)^n dx = \frac{x(ax+b)^{n+1}}{a(n+1)} - \frac{(ax+b)^{n+1}}{a(n+1)(n+2)} + C, n \neq -1, -2$$

$$(2.3) \quad \int \frac{1}{ax+b} dx = \frac{1}{a} \ln |ax+b| + C$$

$$(2.4) \quad \int \frac{1}{(x+a)^2} dx = -\frac{1}{x+a} + C$$

$$(2.5) \quad \int \frac{x}{x+a} dx = \frac{a}{x+a} + \ln |x+a| + C$$

$$(2.6) \quad \int \frac{x}{ax+b} dx = \frac{x}{a} - \frac{b}{a^2} \ln |ax+b| + C$$

$$(2.7) \quad \int \frac{x^2}{ax+b} dx = \frac{x^2}{2a} - \frac{bx}{a^2} + \frac{b^2}{a^3} \ln |ax+b| + C$$

$$(2.8) \quad \int \frac{x}{(ax+b)^2} dx = \frac{1}{a^2} \left( \ln |ax+b| + \frac{b}{ax+b} \right) + C$$

$$(2.9) \quad \int \frac{x^2}{(ax+b)^2} dx = \frac{x}{a^2} - \frac{b^2}{a^3(ax+b)} - \frac{2b}{a^3} \ln |ax+b| + C$$

$$(2.10) \quad \int \frac{x}{(ax+b)^n} dx = \frac{1}{a^2} \left[ \frac{b}{(n-1)(ax+b)^{n-1}} - \frac{1}{(n-2)(ax+b)^{n-2}} \right] + C, n \neq 1, 2$$

$$(2.11) \quad \int \frac{x^2}{(ax+b)^3} dx = \frac{1}{a^3} \left[ \frac{2b}{ax+b} - \frac{b^2}{2(ax+b)^2} + \ln|ax+b| \right] + C$$

$$(2.12) \quad \int \frac{x^2}{(ax+b)^n} dx = \frac{1}{a^3} \left[ \frac{2b}{(n-2)(ax+b)^{n-2}} - \frac{1}{(n-3)(ax+b)^{n-3}} - \frac{b^2}{(n-1)(ax+b)^{n-1}} \right] + C, n \neq 1, 2, 3$$

$$(2.13) \quad \int \frac{1}{a^2+x^2} dx = \frac{1}{a} \arctan\left(\frac{x}{a}\right) + C$$

$$(2.14) \quad \int \frac{x}{a^2+x^2} dx = \frac{1}{2} \ln(a^2+x^2) + C$$

$$(2.15) \quad \int \frac{x^2}{a^2+x^2} dx = x - a \arctan\left(\frac{x}{a}\right) + C$$

$$(2.16) \quad \int \frac{x^3}{a^2+x^2} dx = \frac{1}{2} x^2 - \frac{1}{2} a^2 \ln(a^2+x^2) + C$$

$$(2.17) \quad \int \frac{1}{x(ax+b)} dx = \frac{1}{b} \ln \left| \frac{x}{ax+b} \right| + C$$

$$(2.18) \quad \int \frac{1}{x^2(ax+b)} dx = \frac{a}{b^2} \ln \left| \frac{ax+b}{x} \right| - \frac{1}{bx} + C$$

$$(2.19) \quad \int \frac{1}{x(ax+b)^2} dx = \frac{1}{b(ax+b)} + \frac{1}{b^2} \ln \left| \frac{x}{ax+b} \right| + C$$

$$(2.20) \quad \int \frac{1}{x^2(ax+b)^2} dx = \frac{2a}{b^3} \ln \left| \frac{ax+b}{x} \right| - \frac{2ax+b}{b^2 x(ax+b)} + C$$

$$(2.21) \quad \int \frac{1}{(ax+b)(cx+d)} dx = \frac{1}{ad-bc} \ln \left| \frac{ax+b}{cx+d} \right| + C$$

$$(2.22) \quad \int \frac{x}{(ax+b)(cx+d)} dx = \frac{1}{ad-bc} \left[ \frac{d}{c} \ln|cx+d| - \frac{b}{a} \ln|ax+b| \right] + C$$

$$(2.23) \quad \int \frac{1}{ax^2+bx+c} dx = \begin{cases} \frac{2}{\sqrt{4ac-b^2}} \arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right) + C, b^2 < 4ac \\ \frac{1}{\sqrt{b^2-4ac}} \ln \left| \frac{2cu+b-\sqrt{b^2-4ac}}{2cu+b+\sqrt{b^2-4ac}} \right| + C, b^2 > 4ac \end{cases}, b^2 \neq 4ac$$

$$(2.24)$$

$$\int \frac{x}{ax^2+bx+c} dx = \frac{1}{2a} \ln|ax^2+bx+c| - \begin{cases} \frac{b}{a\sqrt{4ac-b^2}} \arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right) + C, b^2 < 4ac \\ \frac{1}{\sqrt{b^2-4ac}} \ln \left| \frac{2cu+b-\sqrt{b^2-4ac}}{2cu+b+\sqrt{b^2-4ac}} \right| + C, b^2 > 4ac \end{cases}, b^2 \neq 4ac$$

$$(2.25) \quad \int \frac{1}{a^2-x^2} dx = \frac{1}{2a} \ln \left| \frac{a+x}{a-x} \right| + C$$

$$(2.26) \quad \int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + C$$

$$(2.27) \quad \int \frac{1}{(a^2 \pm x^2)^n} dx = \frac{1}{2a^2(n-1)} \left[ \frac{x}{(a^2 \pm x^2)^{n-1}} + (2n-3) \int \frac{1}{(a^2 \pm x^2)^{n-1}} dx \right] + C, n \neq -1$$

$$(2.28) \quad \int \frac{x^n}{x^2 \pm 1} dx = \frac{1}{n-1} x^{n-1} \mp \int \frac{x^{n-2}}{x^2 \pm 1} dx + C, n \neq 0, 1$$

$$(2.29) \quad \int \frac{dx}{x(x^2 \pm a^2)} = \pm \frac{1}{2a^2} \ln \left| \frac{x^2}{a^2 \pm x^2} \right| + C$$

### Radical Functions

$$(3.1) \quad \int \sqrt{ax+b} dx = \frac{2}{3a} (ax+b)^{3/2} + C$$

$$(3.2) \quad \int \frac{1}{\sqrt{ax+b}} dx = \frac{2}{a} \sqrt{ax+b} + C$$

$$(3.3) \quad \int (ax+b)^{(2n+1)/2} dx = \frac{2}{a(2n+3)} (ax+b)^{(2n+3)/2} + C$$

$$(3.4) \quad \int x\sqrt{ax+b} dx = \frac{2x}{5a} (ax+b)^{3/2} - \frac{4b}{15a^2} (ax+b)^{5/2} + C$$

$$(3.5) \quad \int x^2 \sqrt{ax+b} dx = \frac{2x^2}{3a} (ax+b)^{3/2} + \frac{8x}{15a^2} (ax+b)^{5/2} + \frac{16}{105a^3} (ax+b)^{7/2} + C$$

$$(3.6) \quad \int x^m (ax+b)^{(2n+1)/2} dx = \frac{2}{a(2n+3)} \left[ x^m (ax+b)^{(2n+3)/2} - m \int x^{m-1} (ax+b)^{(2n+3)/2} \right] + C$$

$$(3.7) \quad \int \frac{x}{\sqrt{ax+b}} dx = \frac{2x}{a} \sqrt{ax+b} - \frac{4b}{3a^2} (ax+b)^{3/2} + C$$

$$(3.8) \quad \int \frac{x^2}{\sqrt{ax+b}} dx = \frac{2}{15a^3} (2a^2x^2 - 4abx + 18b^2) \sqrt{ax+b} + C$$

$$(3.9) \quad \int \frac{x^n}{\sqrt{ax+b}} dx = \frac{2}{(2n+1)a} \left[ x^n \sqrt{ax+b} - nb \int \frac{x^{n-1}}{\sqrt{ax+b}} dx \right] + C$$

$$(3.10) \quad \int \frac{1}{x\sqrt{ax+b}} dx = \begin{cases} \frac{1}{\sqrt{b}} \ln \left| \frac{\sqrt{ax+b} - \sqrt{b}}{\sqrt{ax+b} + \sqrt{b}} \right| + C, a > 0 \\ \frac{2}{\sqrt{-b}} \arctan \sqrt{\frac{ax+b}{-b}} + C, a < 0 \end{cases}$$

$$(3.11) \quad \int \frac{1}{x^n \sqrt{ax+b}} dx = \frac{1}{a(1-n)} \left[ \frac{\sqrt{ax+b}}{x^{n-1}} + \frac{2(n-3)a}{2} \int \frac{1}{x^{n-1} \sqrt{ax+b}} dx \right] n \neq 1$$

$$(3.12) \quad \int \frac{\sqrt{ax+b}}{x} dx = 2\sqrt{ax+b} + b \begin{cases} \frac{1}{\sqrt{b}} \ln \left| \frac{\sqrt{ax+b} - \sqrt{b}}{\sqrt{ax+b} + \sqrt{b}} \right| + C, a > 0 \\ \frac{2}{\sqrt{-b}} \arctan \sqrt{\frac{ax+b}{-b}} + C, a < 0 \end{cases}$$

$$(3.13) \quad \int \frac{\sqrt{ax+b}}{x^n} dx = \frac{1}{b(1-n)} \left[ \frac{(ax+b)^{3/2}}{x^{n-1}} + \frac{(2n-5)a}{2} \int \frac{1}{x^{n-1}\sqrt{ax+b}} dx \right] + C, n \neq 1$$

$$(3.14) \quad \int \sqrt{\frac{x}{a-x}} dx = \frac{a}{2} \arcsin \left( \frac{a-2x}{a} \right) + \sqrt{x(a-x)} + C$$

$$(3.15) \quad \int \sqrt{\frac{x}{x+a}} dx = \sqrt{x(x+a)} - \frac{a}{2} \ln \left| \sqrt{x(a+x)} + \left( x + \frac{a}{2} \right) \right| + C$$

$$(3.16) \quad \int \sqrt{\frac{x-a}{x+a}} dx = \sqrt{x^2 - a^2} - a \ln \left| \sqrt{x^2 - a^2} + x \right| + C$$

$$(3.17) \quad \int \sqrt{\frac{x+a}{x+b}} dx = \sqrt{(x+a)(x+b)} + (a-b) \ln \left| \sqrt{x+a} + \sqrt{x+b} \right| + C$$

$$(3.18) \quad \int \frac{1}{\sqrt{(x+a)(x+b)}} dx = \ln \left| \frac{a+b}{2} + x + \sqrt{(x+a)(x+b)} \right| + C$$

$$(3.19) \quad \int \sqrt{x(ax+b)} dx = \frac{x\sqrt{x(ax+b)}}{2} + \frac{b}{4a} \sqrt{x(ax+b)} - \frac{b^2}{4a^{3/2}} \ln \left| \sqrt{ax} + \sqrt{ax+b} \right| + C$$

$$(3.20) \quad \int \sqrt{ax^2 + bx + c} dx = \frac{2ax+b}{4a} \sqrt{ax^2 + bx + c} - \frac{b^2 - 4ac}{8a^{3/2}} \ln \left| 2ax + b + 2\sqrt{a(ax^2 + bx + c)} \right| + C$$

$$(3.21)$$

$$\int x\sqrt{ax^2 + bx + c} dx = \frac{\sqrt{ax^2 + bx + c}}{24a^2} - \frac{4ax^2 + bx + 4c}{24a^{3/2}} + \frac{b(b^2 - 4ac)}{16a^{3/2}} \ln \left| b + 2ax + 2\sqrt{a(ax^2 + bx + c)} \right| + C$$

$$(3.22) \quad \int \frac{1}{\sqrt{ax^2 + bx + c}} dx = \frac{1}{\sqrt{a}} \ln \left| 2ax + b + 2\sqrt{a(ax^2 + bx + c)} \right| + C$$

$$(3.23) \quad \int \frac{x}{\sqrt{ax^2 + bx + c}} dx = \frac{1}{a} \sqrt{ax^2 + bx + c} - \frac{b}{2a\sqrt{a}} \ln \left| 2ax + b + 2\sqrt{a(ax^2 + bx + c)} \right| + C$$

$$(3.24) \quad \int \sqrt{2ax - x^2} dx = \frac{x-a}{2} \sqrt{2ax - x^2} + \frac{a^2}{2} \arcsin \left( \frac{x-a}{2} \right) + C, a > 0$$

$$(3.25) \quad \int \frac{dx}{\sqrt{2ax - x^2}} = \arcsin \left( \frac{x-a}{a} \right) + C, a > 0$$

$$(3.26) \quad \int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} x\sqrt{x^2 \pm a^2} \pm \frac{1}{2} a^2 \ln \left| x + \sqrt{x^2 \pm a^2} \right| + C$$

$$(3.27) \quad \int \sqrt{a^2 - x^2} dx = \frac{1}{2} x\sqrt{a^2 - x^2} + \frac{a^2}{2} \arcsin \left( \frac{x}{a} \right) + C$$

$$(3.28) \quad \int x\sqrt{x^2 \pm a^2} dx = \frac{1}{3}(x^2 \pm a^2)^{3/2} + C$$

$$(3.29) \quad \int x^2\sqrt{x^2 \pm a^2} dx = \frac{x}{8}(2x^2 \pm a^2)\sqrt{x^2 \pm a^2} - \frac{a^4}{8} \ln|x + \sqrt{x^2 \pm a^2}| + C$$

$$(3.30) \quad \int x^2\sqrt{a^2 - x^2} dx = \frac{x}{8}(2x^2 - a^2)\sqrt{a^2 - x^2} + \frac{a^4}{8} \arcsin\left(\frac{x}{a}\right) + C$$

$$(3.31) \quad \int \frac{\sqrt{a^2 \pm x^2}}{x} dx = \sqrt{a^2 \pm x^2} - a \ln \left| \frac{a + \sqrt{a^2 \pm x^2}}{x} \right| + C, a > 0$$

$$(3.32) \quad \int \frac{\sqrt{x^2 - a^2}}{x} dx = \sqrt{x^2 - a^2} - a \operatorname{arcsec} \left| \frac{x}{a} \right| + C$$

$$(3.33) \quad \int \frac{\sqrt{x^2 \pm a^2}}{x^2} dx = -\frac{\sqrt{x^2 \pm a^2}}{x} + \ln|x + \sqrt{x^2 \pm a^2}| + C$$

$$(3.34) \quad \int \frac{\sqrt{a^2 - x^2}}{x^2} dx = -\frac{\sqrt{a^2 - x^2}}{x} - \arcsin\left(\frac{x}{a}\right) + C$$

$$(3.35) \quad \int \frac{1}{\sqrt{x^2 \pm a^2}} dx = \ln|x + \sqrt{x^2 \pm a^2}| + C$$

$$(3.36) \quad \int \frac{1}{\sqrt{a^2 - x^2}} dx = \arcsin\left(\frac{x}{a}\right) + C$$

$$(3.37) \quad \int \frac{x}{\sqrt{x^2 \pm a^2}} dx = \sqrt{x^2 \pm a^2} + C$$

$$(3.38) \quad \int \frac{x}{\sqrt{a^2 - x^2}} dx = -\sqrt{a^2 - x^2} + C$$

$$(3.39) \quad \int \frac{x^2}{\sqrt{x^2 \pm a^2}} dx = \frac{1}{2}x\sqrt{x^2 \pm a^2} \mp \frac{a^2}{2} \ln|x + \sqrt{x^2 \pm a^2}| + C$$

$$(3.40) \quad \int \frac{x^2}{\sqrt{a^2 - x^2}} dx = -\frac{x}{2}\sqrt{a^2 - x^2} - \frac{a^2}{2} \arcsin\left(\frac{x}{a}\right) + C$$

$$(3.41) \quad \int \frac{1}{x\sqrt{x^2 + a^2}} dx = \frac{1}{a} \ln \left| \frac{\sqrt{x^2 + a^2} - a}{x} \right| + C$$

$$(3.42) \quad \int \frac{1}{x\sqrt{x^2 - a^2}} dx = \frac{1}{a} \operatorname{arcsec} \left| \frac{x}{a} \right| + C$$

$$(3.43) \quad \int \frac{1}{x\sqrt{a^2 \pm x^2}} dx = -\frac{1}{a} \ln \left| \frac{a + \sqrt{a^2 \pm x^2}}{|x|} \right| + C$$

$$(3.44) \quad \int \frac{1}{x^2\sqrt{a^2 - x^2}} dx = -\frac{\sqrt{a^2 - x^2}}{a^2 x} + C$$

$$(3.45) \quad \int \frac{1}{x^2 \sqrt{x^2 \pm a^2}} dx = \mp \frac{\sqrt{x^2 \pm a^2}}{a^2 x} + C$$

$$(3.46) \quad \int (x^2 \pm a^2)^{3/2} dx = \frac{x}{8} (2x^2 \pm 5a^2) \sqrt{x^2 \pm a^2} + \frac{3a^4}{8} \ln \left| x + \sqrt{x^2 \pm a^2} \right| + C$$

$$(3.47) \quad \int \frac{1}{(a^2 \pm x^2)^{3/2}} dx = \frac{\pm x}{a^2 \sqrt{a^2 \pm x^2}} + C$$

$$(3.48) \quad \int \frac{x^2}{(x^2 \pm a^2)^{3/2}} dx = \frac{-x}{\sqrt{x^2 \pm a^2}} + \ln \left| x + \sqrt{x^2 \pm a^2} \right| + C$$

$$(3.49) \quad \int x^n (x^2 \pm 1)^{(2k+1)/2} dx = \frac{1}{2k+3} x^{n-1} (x^2 \pm 1)^{(2k+3)/2} - \frac{n-1}{2k+3} \int x^{n-2} (x^2 \pm 1)^{(2k+3)/2} dx + C, n \neq 0$$

$$(3.50) \quad \int x^n (1-x^2)^{(2k+1)/2} dx = -\frac{1}{2k+3} x^{n-1} (1-x^2)^{(2k+3)/2} + \frac{n-1}{2k+3} \int x^{n-2} (1-x^2)^{(2k+3)/2} dx + C, n \neq 0$$

### Trigonometric Functions

$$(4.1) \quad \int \sin(ax) dx = -\frac{1}{a} \cos(ax) + C$$

$$(4.2) \quad \int \sin^2 ax dx = \frac{1}{2} x - \frac{1}{4a} \sin(2ax) + C$$

$$(4.3) \quad \int \sin^3 ax dx = -\frac{1}{3a} \sin^2(ax) \cos(ax) - \frac{2}{3a} \cos(ax) + C$$

$$(4.4) \quad \int \sin^4 ax dx = -\frac{1}{4a} \sin^3(ax) \cos(ax) - \frac{3}{8a} \sin(ax) \cos(ax) + \frac{3}{8} x + C$$

$$(4.5) \quad \int \sin^n x dx = \frac{1}{n} \sin^{n-1} x \cos x + \frac{n-1}{n} \int \sin^{n-2} x dx$$

$$(4.6) \quad \int x \sin(ax) dx = -\frac{1}{a} x \cos(ax) + \frac{1}{a^2} \sin(ax) + C$$

$$(4.7) \quad \int x^2 \sin(ax) dx = \frac{2}{a^3} \cos(ax) + \frac{2}{a^2} \sin(ax) - \frac{1}{a} x^2 \cos(ax) + C$$

$$(4.8) \quad \int x^n \sin x dx = -x^n \cos x + n \int x^{n-1} \cos x dx$$

$$(4.9) \quad \int \cos ax dx = \frac{1}{a} \sin ax + C$$

$$(4.10) \quad \int \cos^2 ax dx = \frac{1}{2} x + \frac{1}{4a} \sin(2ax) + C$$

$$(4.11) \quad \int \cos^3(ax) dx = \frac{1}{3a} \sin(ax) \cos^2(ax) + \frac{2}{3a} \sin(ax) + C$$

$$(4.12) \quad \int \cos^4(ax) dx = \frac{1}{4a} \sin(ax) \cos^3(ax) + \frac{3}{8a} \sin(ax) \cos(ax) + \frac{3}{8} x + C$$

$$(4.13) \quad \int \cos^n x dx = \frac{1}{n} \cos^{n-1} x \sin x + \frac{n-1}{n} \int \cos^{n-2} x dx$$

$$(4.14) \quad \int x \cos(ax) dx = \frac{1}{a^2} \cos(ax) + \frac{1}{a} x \sin(ax) + C$$

$$(4.15) \quad \int x^2 \cos(ax) dx = \frac{2}{a^2} x \cos(ax) + \frac{1}{a} x^2 \sin(ax) - \frac{2}{a^3} \sin(ax) + C$$

$$(4.16) \quad \int x^n \cos x dx = x^n \sin x - n \int x^{n-1} \sin x dx$$

$$(4.17) \quad \int \cos(ax) \sin(bx) dx = \frac{1}{2(a-b)} \cos[(a-b)x] - \frac{1}{2(a+b)} \cos[(a+b)x] + C$$

$$(4.18) \quad \int \sin(ax) \sin(bx) dx = \frac{1}{2} \left[ \frac{\sin(a-b)x}{a-b} - \frac{\sin(a+b)x}{a+b} \right] + C$$

$$(4.19) \quad \int \cos(ax) \cos(bx) dx = \frac{1}{2} \left[ \frac{\sin(a+b)x}{a+b} + \frac{\sin(a-b)x}{a-b} \right] + C$$

$$(4.20) \quad \int \sin^2(ax) \cos(bx) dx = -\frac{1}{4(2a-b)} \sin[(2a-b)x] + \frac{1}{2b} \sin(bx) - \frac{1}{4(2a+b)} \sin[(2a+b)x] + C$$

$$(4.21) \quad \int \sin^2(ax) \cos(ax) dx = \frac{1}{3a} \sin^3(ax) + C$$

$$(4.22) \quad \int \cos^2(ax) \sin(ax) dx = -\frac{1}{3a} \cos^3(ax) + C$$

$$(4.23) \quad \int \cos^2(ax) \sin(bx) dx = \frac{1}{4(2a-b)} \cos[(2a-b)x] - \frac{1}{2b} \cos(bx) - \frac{1}{4(2a+b)} \cos[(2a+b)x] + C$$

$$(4.24)$$

$$\int \sin^2(ax) \cos^2(bx) dx = \frac{1}{4} x - \frac{1}{8a} \sin(2ax) - \frac{1}{16(a-b)} \sin[2(a-b)x] + \frac{1}{8b} \sin(2bx) - \frac{1}{16(a+b)} \sin[2(a+b)x] + C$$

$$(4.25) \quad \int \sin^2(ax) \cos^2(ax) dx = \frac{1}{8} x - \frac{1}{32a} \sin(4ax) + C$$

$$(4.26) \quad \int \sin^m x \cos^n x = \begin{cases} -\frac{\sin^{m-1} x \cos^{n+1} x}{m+n} + \frac{m-1}{m+n} \int \sin^{m-2} x \cos^n x dx, m \neq -n \\ \frac{\sin^{m+1} x \cos^{n-1} x}{m+n} + \frac{n-1}{m+n} \int \sin^m x \cos^{n-2} x dx, m \neq -n \end{cases}$$

$$(4.27) \quad \int \frac{1}{1 \pm \sin x} dx = \tan x \mp \sec x + C$$

$$(4.28) \quad \int \frac{dx}{1 \pm \sin ax} = \mp \frac{1}{a} \tan \left( \frac{\pi}{4} \mp \frac{ax}{2} \right) + C$$

$$(4.29) \quad \int \frac{1}{\sin x \cos x} dx = \int \csc x \sec x dx = \ln |\tan x| + C$$

$$(4.30) \quad \int \frac{1}{1 \pm \cos x} dx = -\cot x \pm \csc x + C$$

$$(4.31) \quad \int \frac{dx}{1 + \cos ax} = \frac{1}{a} \tan\left(\frac{ax}{2}\right) + C$$

$$(4.32) \quad \int \frac{dx}{1 - \cos ax} = -\frac{1}{a} \cot\left(\frac{ax}{2}\right) + C$$

$$(4.33) \quad \int \tan ax dx = -\frac{1}{a} \ln|\cos ax| + C$$

$$(4.34) \quad \int \tan^2 ax dx = \frac{1}{a} \tan ax - x + C$$

$$(4.35) \quad \int \tan^3(ax) dx = \frac{1}{a} \ln|\cos(ax)| + \frac{1}{2a} \sec^2(ax) + C$$

$$(4.36) \quad \int \tan^n x dx = \frac{1}{n-1} \tan^{n-1} x - \int \tan^{n-2} x dx, n \neq 1$$

$$(4.37) \quad \int \frac{1}{1 \pm \tan x} dx = \frac{1}{2} [x \pm \ln|\cos x \pm \sin x|] + C$$

$$(4.38) \quad \int \sec(ax) dx = \frac{1}{a} \ln|\sec(ax) + \tan(ax)| + C = \frac{2}{a} \tanh^{-1}\left(\tan \frac{x}{2}\right) + C$$

$$(4.39) \quad \int \sec^2(ax) dx = \frac{1}{a} \tan(ax) + C$$

$$(4.40) \quad \int \sec^3 x dx = \frac{1}{2} \sec x \tan x + \frac{1}{2} \ln|\sec x + \tan x| + C$$

$$(4.41) \quad \int \sec^n x dx = \frac{1}{n-1} \sec^{n-2} x \tan x + \frac{n-2}{n-1} \int \sec^{n-2} x dx, n \neq 1$$

$$(4.42) \quad \int \sec^2(ax) \tan(ax) dx = \frac{1}{2a} \sec^2(ax) + C$$

$$(4.43) \quad \int \sec^n(ax) \tan(ax) dx = \frac{1}{na} \sec^n(ax) + C, n \neq 0$$

$$(4.44) \quad \int \frac{1}{1 \pm \sec x} dx = x + \cot x \mp \csc x + C$$

$$(4.45) \quad \int \cot^2(x) dx = -x - \cot x + C$$

$$(4.46) \quad \int \cot^n x dx = -\frac{1}{n-1} \cot^{n-1} x - \int \cot^{n-2} x dx, n \neq 1$$

$$(4.47) \quad \int \frac{1}{1 \pm \cot x} dx = \frac{1}{2} [x \mp \tan x \pm \sec x] + C$$

$$(4.48) \quad \int \csc(ax) dx = \frac{1}{a} \ln \left| \tan \frac{ax}{2} \right| = \frac{1}{a} \ln |\csc(ax) - \cot(ax)| + C$$

$$(4.49) \quad \int \csc^2(ax) dx = -\frac{1}{a} \cot(ax) + C$$

$$(4.50) \quad \int \csc^3 x dx = \frac{1}{2} \ln|\csc x - \cot x| - \frac{1}{2} \csc x \cot x + C$$

$$(4.51) \quad \int \csc^n x dx = -\frac{1}{n-1} \csc^{n-2} x \cot x + \frac{n-2}{n-1} \int \csc^{n-2} x dx, n \neq 1$$

$$(4.52) \quad \int \csc^n(ax) \cot(ax) dx = -\frac{1}{na} \csc^n(ax) + C, n \neq 0$$

$$(4.53) \quad \int \frac{1}{1 \pm \csc x} dx = x - \tan x \pm \sec x + C$$

### Inverse Trigonometric Functions

$$(5.1) \quad \int \arcsin x dx = x \arcsin x + \sqrt{1-x^2} + C$$

$$(5.2) \quad \int x \arcsin x dx = \frac{1}{2} x^2 \arcsin x - \frac{1}{4} \arcsin x + \frac{1}{4} x \sqrt{1-x^2} + C$$

$$(5.3) \quad \int x^n \arcsin x dx = \frac{1}{n+1} x^{n+1} \arcsin x - \frac{1}{n+1} \int \frac{x^{n+1}}{\sqrt{1-x^2}} dx + C, n \neq -1$$

$$(5.4) \quad \int \arccos x dx = x \arccos x - \sqrt{1-x^2} + C$$

$$(5.5) \quad \int x \arccos x dx = \frac{1}{2} x^2 \arccos x + \frac{1}{4} x \sqrt{1-x^2} + \frac{1}{4} \arcsin x + C$$

$$(5.6) \quad \int \arctan x dx = x \arctan x - \ln \sqrt{1+x^2} + C$$

$$(5.7) \quad \int x \arctan x dx = \frac{1}{2} x^2 \arctan x + \frac{1}{2} \arctan x - \frac{1}{2} x + C$$

$$(5.8) \quad \int x^n \arctan x dx = \frac{1}{n+1} x^{n+1} \arctan x - \frac{1}{n+1} \int \frac{x^{n+1}}{x^2+1} dx + C, n \neq -1$$

$$(5.9) \quad \int \operatorname{arccot} x dx = x \operatorname{arccot} x + \ln \sqrt{1+x^2} + C$$

$$(5.10) \quad \int x \operatorname{arccot} x dx = \frac{1}{2} x^2 \operatorname{arccot} x + \frac{1}{2} x + \frac{1}{2} \operatorname{arccot} x + C$$

$$(5.11) \quad \int \operatorname{arcsec} x dx = x \operatorname{arcsec} x - \ln \left| x + \sqrt{x^2-1} \right| + C$$

$$(5.12) \quad \int x \operatorname{arcsec} x dx = \frac{1}{2} x^2 \operatorname{arcsec} x - \frac{1}{2} \sqrt{x^2-1} + C$$

$$(5.13) \quad \int x^n \operatorname{arcsec} x dx = \frac{1}{n+1} x^{n+1} \operatorname{arcsec} x - \frac{1}{n+1} \int \frac{x^n}{\sqrt{x^2-1}} dx + C, n \neq -1$$

$$(5.14) \quad \int \operatorname{arccsc} x dx = x \operatorname{arccsc} x + \ln \left| x + \sqrt{x^2-1} \right| + C$$

$$(5.15) \quad \int x \operatorname{arccsc} x dx = \frac{1}{2} x^2 \operatorname{arccsc} x + \frac{1}{2} \sqrt{x^2-1} + C$$

### Hyperbolic Trigonometric Functions

$$(6.1) \quad \int \sinh(ax) dx = \frac{1}{a} \cosh(ax) + C$$

$$(6.2) \quad \int \cosh(ax) dx = \frac{1}{a} \sinh(ax) + C$$

$$(6.3) \quad \int \tanh(ax) dx = \frac{1}{a} \ln |\cosh(ax)| + C$$

$$(6.4) \quad \int \coth(ax) dx = \frac{1}{a} \ln |\sinh(ax)| + C$$

$$(6.5) \quad \int \operatorname{sech}(ax) dx = \frac{2}{a} \arctan(e^{ax}) + C$$

$$(6.6) \quad \int \operatorname{csch}(ax) dx = \frac{1}{a} \ln \left| \frac{e^{ax} - 1}{e^{ax} + 1} \right| + C$$

$$(6.7) \quad \int x \sinh(ax) dx = \frac{1}{a} x \cosh(ax) - \frac{1}{a^2} \sinh(ax) + C$$

$$(6.8) \quad \int x \cosh(ax) dx = \frac{1}{a} x \sinh(ax) - \frac{1}{a^2} \cosh(ax) + C$$

$$(6.9) \quad \int x^n \sinh(ax) dx = \frac{1}{a} x^n \cosh(ax) - \frac{n}{a} \int x^{n-1} \cosh(ax) dx + C$$

$$(6.10) \quad \int x^n \cosh(ax) dx = \frac{1}{a} x^n \sinh(ax) - \frac{n}{a} \int x^{n-1} \sinh(ax) dx + C$$

$$(6.11) \quad \int \sinh(ax) \cosh(ax) dx = \frac{1}{4a} \sinh(2ax) - \frac{1}{2} x + C$$

$$(6.12) \quad \int \sinh(ax) \cosh(bx) dx = \frac{1}{b^2 - a^2} [b \sinh(ax) \cosh(bx) - a \sinh(bx) \cosh(ax)] + C$$

$$(6.13) \quad \int \sinh(ax) \sin(bx) dx = \frac{1}{a^2 + b^2} [a \cosh(ax) \sin(bx) - b \sinh(bx) \cos(ax)] + C$$

$$(6.14) \quad \int \sinh(ax) \cos(bx) dx = \frac{1}{a^2 + b^2} [a \cosh(ax) \cos(bx) + b \sinh(ax) \sin(bx)] + C$$

$$(6.15) \quad \int \cosh(ax) \sin(bx) dx = \frac{1}{a^2 + b^2} [a \sinh(ax) \sin(bx) - b \cosh(ax) \cos(bx)] + C$$

$$(6.16) \quad \int \cosh(ax) \cos(bx) dx = \frac{1}{a^2 + b^2} [b \cosh(ax) \sin(bx) + a \sinh(ax) \cos(bx)] + C$$

### Logarithmic Functions

$$(7.1) \quad \int \ln(ax) dx = x \ln(ax) - x + C$$

$$(7.2) \quad \int \frac{\ln(ax)}{x} dx = \frac{1}{2} \ln^2(ax) + C$$

$$(7.3) \quad \int x^n \ln x dx = -\frac{1}{(n+1)^2} x^{n+1} + \frac{1}{n+1} x^{n+1} \ln x + C, n \neq -1$$

$$(7.4) \quad \int \ln(ax+b)dx = \left(x + \frac{b}{a}\right)\ln(ax+b) - x, a \neq 0$$

$$(7.5) \quad \int \ln(x^2 + a^2)dx = x \ln(x^2 + a^2) + 2a \arctan\left(\frac{x}{a}\right) - 2x + C$$

$$(7.6) \quad \int \ln(x^2 - a^2)dx = x \ln(x^2 - a^2) + a \ln\left|\frac{x+a}{x-a}\right| - 2x + C$$

$$(7.7) \quad \int \ln(ax^2 + bx + c)dx = \frac{1}{a}\sqrt{4ac - b^2} \arctan\left(\frac{2ax + b}{\sqrt{4ac - b^2}}\right) - 2x + \left(\frac{b}{2a} + x\right)\ln(ax^2 + bx + c) + C$$

$$(7.8) \quad \int x \ln(ax+b)dx = \frac{b}{2a}x - \frac{1}{4}x^2 + \frac{1}{2}x^2 \ln(ax+b) - \frac{b^2}{2a^2}\ln(ax+b) + C$$

$$(7.9) \quad \int x \ln(a^2 - b^2x^2)dx = -\frac{1}{2}x^2 + \frac{1}{2}x^2 \ln(a^2 - b^2x^2) - \frac{a^2}{2b^2}\ln(ax+b) + C$$

$$(7.10) \quad \int \ln^2 x dx = 2x - 2x \ln x + x \ln^2 x + C$$

$$(7.11) \quad \int \ln^n x dx = x \ln^n x - n \int \ln^{n-1} x dx + C$$

$$(7.12) \quad \int x^n \ln^m x dx = \frac{1}{n+1}x^{n+1} \ln^m x - \frac{m}{n+1} \int x^n \ln^{m-1} x dx + C, m, n \neq -1$$

$$(7.13) \quad \int \frac{1}{x \ln x} dx = \ln|\ln x| + C$$

$$(7.14) \quad \int \log_a b x dx = \frac{1}{\ln a} [x \ln b x - x] + C$$

### Exponential Functions

$$(8.1) \quad \int e^{ax} dx = \frac{1}{a}e^{ax} + C$$

$$(8.2) \quad \int x e^{ax} dx = \frac{1}{a}x e^{ax} - \frac{1}{a^2}e^{ax} + C = \frac{ax-1}{a^2}e^{ax} + C$$

$$(8.3) \quad \int x^2 e^{ax} dx = \frac{a^2 x^2 - 2ax + 2}{a^3}e^{ax} + C$$

$$(8.4) \quad \int x^3 e^{ax} dx = \frac{a^3 x^3 - 3a^2 x^2 + 6ax - 6}{a^4}e^{ax} + C$$

$$(8.5) \quad \int x^n e^{ax} dx = \frac{1}{a}x^n e^{ax} - \frac{n}{a} \int x^{n-1} e^{ax} dx + C$$

$$(8.6) \quad \int \frac{e^{ax}}{x^n} dx = -\frac{e^{ax}}{(n-1)x^{n-1}} + \frac{a}{n-1} \int \frac{e^{ax}}{x^{n-1}} dx + C, n \neq 1$$

$$(8.7) \quad \int x e^{-ax^2} dx = -\frac{1}{2a}e^{-ax^2} + C$$

$$(8.8) \quad \int x^3 e^{ax^2} dx = \frac{1}{2a} x^2 e^{ax^2} - \frac{1}{2a^2} e^{ax^2} + C$$

$$(8.9) \quad \int x^{2n+1} e^{ax^2} dx = \frac{1}{2a} x^{2n} e^{ax^2} - \frac{n}{a} \int x^{2n-1} e^{ax^2} dx + C, n \geq 0$$

$$(8.10) \quad \int \frac{1}{a + be^{kx}} dx = \frac{1}{ak} [kx - \ln|a + be^{kx}|] + C$$

$$(8.11) \quad \int e^{ax} \sin(bx) dx = \frac{e^{ax}}{a^2 + b^2} [a \sin(bx) - b \cos(bx)] + C$$

$$(8.12) \quad \int e^{ax} \cos(bx) dx = \frac{e^{ax}}{a^2 + b^2} [b \sin(bx) + a \cos(bx)] + C$$

$$(8.13) \quad \int x e^x \sin x dx = \frac{1}{2} e^x [\cos x - x \cos x + x \sin x] + C$$

$$(8.14) \quad \int x e^x \cos x dx = \frac{1}{2} e^x [x \cos x + x \sin x - \sin x] + C$$

$$(8.15) \quad \int e^{ax} \sinh(bx) dx = \begin{cases} \frac{e^{ax}}{a^2 - b^2} [a \sinh(bx) - b \cosh(bx)] + C, a^2 \neq b^2 \\ \frac{1}{4a} e^{2ax} - \frac{x}{2} + C, a = b \end{cases}$$

$$(8.16) \quad \int e^{ax} \cosh(bx) dx = \begin{cases} \frac{e^{ax}}{a^2 - b^2} [a \cosh(bx) - b \sinh(bx)] + C, a^2 \neq b^2 \\ \frac{1}{4a} e^{2ax} + \frac{x}{2} + C, a = b \end{cases}$$

$$(8.17) \quad \int a^{bx} dx = \frac{1}{b \ln a} a^{bx} + C, a > 0, a \neq 1$$