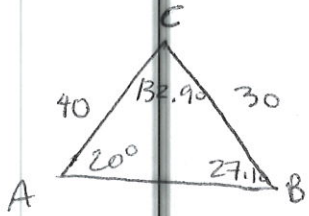
MAT 166, Exam #4, Fall 2017 Name KEY

Instructions: Show all work. Give exact answers unless specifically asked to round. Complete all parts of each question. Questions that provide only answers and no work will not receive full credit. If you use your calculator (only when problems don't instruct you to do the problem by hand), showing calculator steps will count as "work".

1. For each situation below, find all the missing elements of the given triangle. Be sure to check if two triangles exist, and if so, find both. If no triangle exists, explain why. Round lengths and angles (in degrees) to the nearest tenth.

a. $a = 30, b = 40, A = 20°$

$\frac{\sin 20°}{30} = \frac{\sin B}{40}$ $B \Rightarrow 27.1°$



$C = 132.9°$ $\frac{30}{\sin 20°} = \frac{c}{\sin 132.9°}$ $c = 64.3$

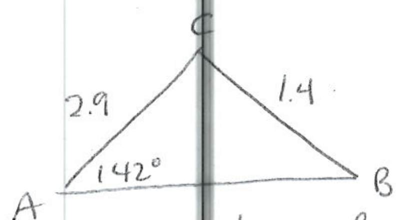
2 triangles

$B = 152.9°$ $\frac{30}{\sin 20°} = \frac{c}{\sin 7.1°}$ $c = 10.8$

$C = 7.1°$

b. $a = 1.4, b = 2.9, A = 142°$

$\frac{\sin 142°}{1.4} = \frac{\sin B}{2.9}$

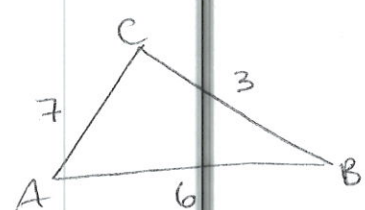


$\sin B = 1.275...$

no triangle

c. $a = 3, b = 7, c = 6$

$\cos C = \frac{6^2 - 7^2 - 3^2}{-2ab} = -2(7)(3)$



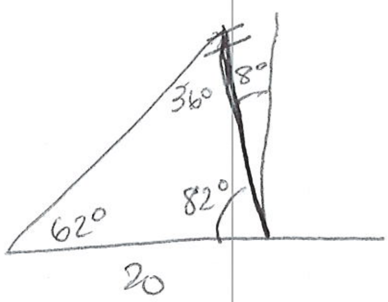
$C = 58.4°$

$\cos B = \frac{7^2 - 6^2 - 3^2}{-2(6)(3)}$

$A = 25.2°$

$B = 96.4°$

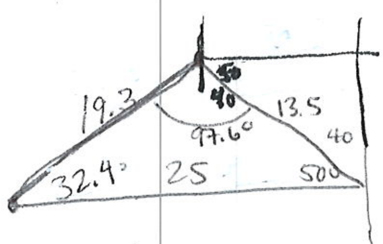
2. When the angle of elevation of the sun is 62°, a telephone pole that is tilted at an angle of 8° away from the sum casts a shadow 20 feet long. Determine the length of the pole to the nearest foot.



$$\frac{20}{\sin 36^\circ} = \frac{b}{\sin 62^\circ}$$

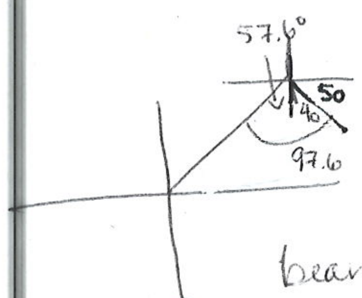
$b = 30.\text{ft.}$

3. You are on a fishing boat that leaves its pier and heads east. After traveling for 25 miles, there is a report of rough seas directly north, so the captain turns the boat to a bearing of $N40°W$ for 13.5 miles. How far is the boat to the pier, and in which direction would the boat have to sail in order to read port from their current position?



$c^2 = 25^2 + 13.5^2 - 2(25)(13.5)\cos 50^\circ$

$c =$ 19.3



$$\frac{\sin B}{13.5} = \frac{\sin 50^\circ}{19.3}$$

$B = 32.4$

$C = 97.6$

bearing
S57.6°W

4. Convert the equation $y^2 = 12x$ into polar coordinates.

$r^{\cancel{2}} \sin^2\theta = 12\cancel{r}\cos\theta$

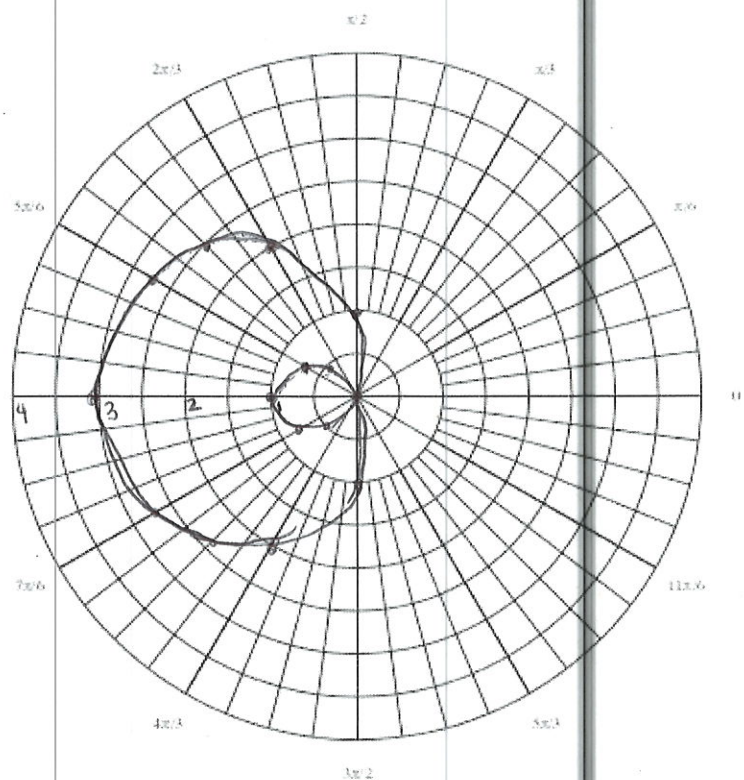
$r = 12\cot\theta \csc\theta$

5. Convert the equation $r^2 \sin 2\theta = 6$ into rectangular coordinates.

$$2r^2 \sin\theta \cos\theta = 6$$
$$r \sin\theta \, r\cos\theta = 3$$
$$yx = 3$$
$$y = \frac{3}{x}$$

6. Graph the polar equation $r = 1 - 2\cos\theta$ on the polar graph below. Clearly label at least 6 points and show work.

θ	r
0	-1
$\pi/6$	$-.7321 = 1-\sqrt{3}$
$\pi/4$	$-.4142 = 1-\sqrt{2}$
$\pi/3$	$0 = 1-1$
$\pi/2$	$1 = 1-0$
$2\pi/3$	$2 = 1+1$
$3\pi/4$	$2.4142 = 1+\sqrt{2}$
$5\pi/6$	$2.7321 = 1+\sqrt{3}$
π	3



7. Find $(-\sqrt{3}+i)^5$ using DeMoivre's Theorem. (You will no receive credit for FOILing.) Write the result in standard form with exact values.

$r = \sqrt{3+1} = \sqrt{4} = 2 \qquad \theta = \tan^{-1}\left(\frac{1}{-\sqrt{3}}\right) = -\pi/6 + \pi = 5\pi/6$

$2(\cos 5\pi/6 + i \sin 5\pi/6)$

$[-\sqrt{3}+i]^5 = [2^5(\cos 25\pi/6 + i \sin 25\pi/6)] =$

$32\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right) = 16\sqrt{3} + 16i$

8. For $z_1 = 4\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$ and $z_2 = 12\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$, find the following products or quotients. For standard angles, fully evaluate and give exact answers. You may leave non-standard angles in polar form.

a. $z_1 z_2$

$$48(\cos 3\pi/2 + i \sin 3\pi/2) = 48(0 - i) = -48i$$

b. $\frac{z_2}{z_1}$

$$\frac{12}{4}(\cos \pi/6 + i \sin \pi/6) = 3(\cos \pi/6 + i \sin \pi/6)$$

$$= 3\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right) = \frac{3\sqrt{3}}{2} + \frac{3}{2}i$$

9. Find all the the 4[th] roots of $-2 - 2\sqrt{3}i$. You may leave your solutions in polar form.

$r = \sqrt{4+12} = \sqrt{16} = 4 \qquad \theta = \tan^{-1}\left(\frac{-2\sqrt{3}}{-2}\right) = \pi/3 + \pi = 4\pi/3$

$\left[4(\cos 4\pi/3 + i \sin 4\pi/3)\right]^{1/4} = \sqrt{2}(\cos \pi/3 + i \sin \pi/3) = \sqrt{2}\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$

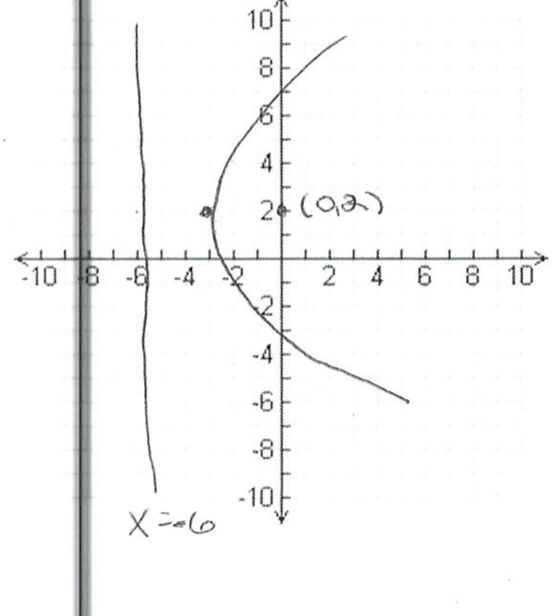
$\left[4(\cos 10\pi/3 + i \sin 10\pi/3)\right]^{1/4} = \sqrt{2}(\cos 5\pi/6 + i \sin 5\pi/6) = \sqrt{2}\left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)$

$\left[4(\cos 16\pi/3 + i \sin 16\pi/3)\right]^{1/4} = \sqrt{2}(\cos 4\pi/3 + i \sin 4\pi/3) = \sqrt{2}\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)$

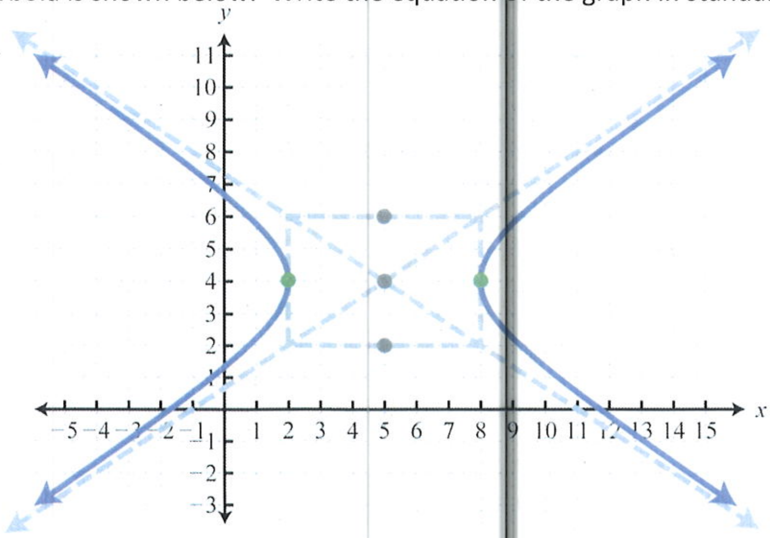
$\left[4(\cos 22\pi/3 + i \sin 22\pi/3)\right]^{1/4} = \sqrt{2}(\cos 11\pi/6 + i \sin 11\pi/6) = \sqrt{2}\left(\frac{\sqrt{3}}{2} - \frac{1}{2}i\right)$

10. Graph the equation $(y-2)^2 = 12(x+3)$ on the axes below. Clearly label the focus, vertex and directrix.

$a = 3$



11. The graph of a hyperbola is shown below. Write the equation of the graph in standard form.



$$\frac{(x-5)^2}{9} - \frac{(y-4)^2}{4} = 1$$

12. An ellipse has the endpoints of the major axis at (7,9) and (7,3), and one focus at (7,8). Find the equation of the ellipse in standard form.

center $(7,6)$

$a = 3$, $c = 2$

$b = \sqrt{3^2 - 2^2} = \sqrt{9-4} = \sqrt{5}$

$$\frac{(x-7)^2}{5} + \frac{(y-6)^2}{9} = 1$$

13. Identify the type of conic given by the equation $x^2 + 6x - 4y + 1 = 0$ and put the equation in standard form.

parabola

$(x^2 + 6x + 9) = 4y - 1 + 9$

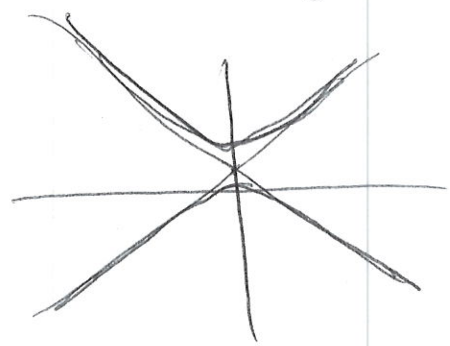
$(x+3)^2 = 4y + 8$

$(x+3)^2 = 4(y+2)$

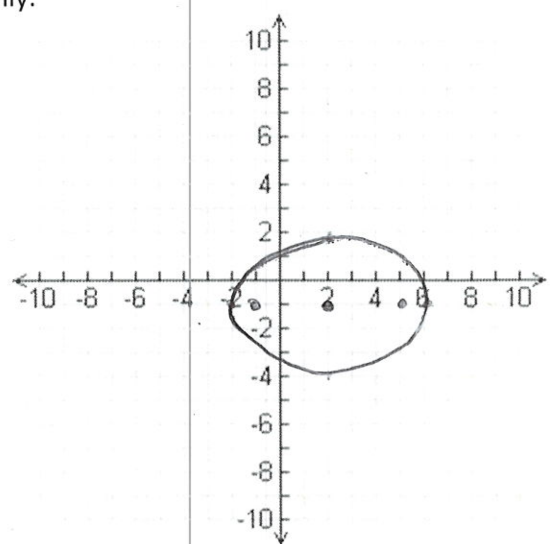
14. Given the equation $r = \frac{8}{2+4\sin\theta}$, determine the type of conic this represents by finding the eccentricity of the graph. Then use technology to sketch the graph and confirm your results.

$$\frac{8 \div \frac{1}{2}}{(2+4\sin\theta)\frac{1}{2}} = \frac{4}{1+2\sin\theta}$$

$e=2$ hyperbola



15. Sketch the graph of the parametric equations $x = 2 + 4\cos t$, $y = -1 + 3\sin t$ by plotting at least 4 points (and labeling them). Then convert the equation back to an equation in x and y only.



center $(2,-1)$

ellipse

$a=4, b=3$

$$\frac{(x-2)^2}{16} + \frac{(y+1)^2}{9} = 1$$