MTH 166 Homework #12 Key

1a. $-1, 3, 7, 11, 15, \ldots$

b. $-4, 5, -6, 7, -8, \ldots$

c. $1/2, -1/3, 1/5, -1/9, 1/17, \ldots$

d. $4, 11, 25, 53, 109, \ldots$

e. $2, 3/2, 8/3, 15/2, 144/5, \ldots$

f. $1, -3, 9, -27, 81, \ldots$

g. $0, 1/2, 6/7, 9/8, 4/3, \ldots$

h. $7, 12, 19, 26, 33, \ldots$

i. $0, 1, 2, 3/2, 2/3, \ldots$

2.a. $5+10+15+20+25+30 = 105$

b. $(-1/3)^2 + (-1/3)^3 + (-1/3)^4 = 1/9 - 1/27 + 1/81 = 7/81$

c. $1+8+27+64+125 = 225$

d. $\frac{-1}{1} + \frac{1}{2} + \frac{-1}{6} + \frac{1}{24} - \frac{1}{120} = \frac{-19}{30}$

3a. $\sum_{n=0}^{15} n^2$

b. $\sum_{n=1}^{14} \frac{n}{n+1}$

c. $\sum_{n=0}^{13} (2n+5)$

d. $\sum_{n=1}^{11} 2^n$

e. $\sum_{i=1}^{n} \frac{i}{9^i}$

4a. $a_n = 5(3)^{n-1}$ or $\frac{5}{3}(3)^n$

b. $24(1/3)^{n-1} = a_n$ or $8(1/3)^n$

c. $a_n = -6(-5)^{n-1}$ or $(-6/5)(-5)^n$

d. $a_n = 1000(-1/2)^{n-1}$ or $(-2000)(-1/2)^n$

5a. $a_n = 3(4)^{n-1}$ starting at $n=1$

b. $5(-1/5)^{n-1} = a_n$

c. $12(-1/2)^{n-1} = a_n$

OR

starting at $n=0$

a. $a_n = 3(4)^n$

b. $a_n = 5(-1/5)^n$

c. $a_n = 12(-1/2)^n$

6. a. for $n=1$ $(1) = 1$ ✓

assume for k, show k+1

$$\sum_{i=1}^{k+1} (2i-1) = \sum_{i=1}^{k} (2i+1) + 2(k+1)-1 = k^2 + 2k + 1 = (k+1)^2$$

which is what the formula predicts. ✓

6b. for $n=1$ $\frac{1}{1(2)} = \frac{1}{2}$ ✓

assume for k, show for k+1

$$\sum_{i=1}^{k+1} \frac{1}{i(i+1)} = \sum_{i=1}^{k} \frac{1}{i(i+1)} + \frac{1}{(k+1)(k+2)} = \frac{k}{k+1} + \frac{1}{(k+1)(k+2)} = \frac{k(k+2)+1}{(k+1)(k+2)} = \frac{k^2+2k+1}{(k+1)(k+2)}$$

$$= \frac{(k+1)^{\cancel{2}}}{\cancel{(k+1)}(k+2)} = \frac{k+1}{k+2}$$ which is what the formula predict ✓

c. $n=1$ $2^1 = 2^2 - 2 = 4-2$ ✓

assume for k, show for k+1

$$\sum_{i=1}^{k+1} 2^i = \sum_{i=1}^{k} 2^i + 2^{k+1} = 2^{k+1} - 2 + 2^{k+1} = (2^{k+1})2 - 2 = 2^{k+2} - 2$$

which is what the formula predicts ✓

d. $n=1$ $1 = \frac{1(2)(3)}{6} = 1$ ✓

assume for k, show for k+1

$$\sum_{i=1}^{k+1} i^2 = \sum_{i=1}^{k} k^2 + (k+1)^2 = \frac{k(k+1)(2k+1)}{6} + (k+1)^2 = \frac{k(k+1)(2k+1)}{6} + \frac{6(k+1)^2}{6}$$

$$= \frac{(k+1)}{6}\left[k(2k+1) + 6(k+1)\right] = \frac{k+1}{6}\left[2k^2 + k + 6k + 6\right] = \frac{k+1}{6}\left[2k^2+7k+6\right]$$

$$= \frac{k+1}{6}\left[(2k+3)(k+2)\right] = \frac{(k+1)(k+2)(2k+3)}{6}$$ which is what the formula predicts ✓

7a. $x^3 + 3x^2(2) + 3x(4) + 8 = x^3 + 6x^2 + 12x + 8$

b. $(2x^3)^4 + 4(2x^3)^3(1) + 6(2x^3)^2(1)^2 + 4(2x^3)(1)^3 + 1$

$= 16x^{12} + 32x^9 + 24x^6 + 8x^3 + 1$

c. $(5x)^4 + 4(5x)^3(-1) + 6(5x)^2(-1)^2 + 4(5x)(-1)^3 + (-1)^4$

$625x^4 - 500x^3 + 150x^2 - 20x + 1$

d. $(3x)^5 + 5(3x)^4(-y) + 10(3x)^3(-y)^2 + 10(3x)^2(-y)^3 + 5(3x)(-y)^4 + (-y)^5$

$= 243x^5 - 405x^4y + 270x^3y^2 - 90x^2y^3 + 15xy^4 - y^5$

8a. $(2x+y)^6$ $n=2$ (third term since starts at 0)

$\binom{6}{2}(2x)^4(y)^2 = 15(16x^4)y^2 = 240x^4y^2$

8b. $(x - \frac{1}{2})^{22}$ $n = 14$

$$\binom{22}{14}(x)^8\left(-\frac{1}{2}\right)^{14} = 319770x^8\left(\frac{1}{16384}\right) = \frac{159885}{8192}x^8$$

c. $(x^2+y^2)^9$ $n = 3$

$$\binom{9}{3}(x^2)^6(y^2)^3 = 84x^{12}y^6$$

d. $(x^3+x^{-2})^4$ $n=2$

$$\binom{4}{2}(x^3)^2(x^{-2})^2 = 6x^6x^{-4} = 6x^2$$