MAT 166, Homework #9, Fall 2017 Name KEY

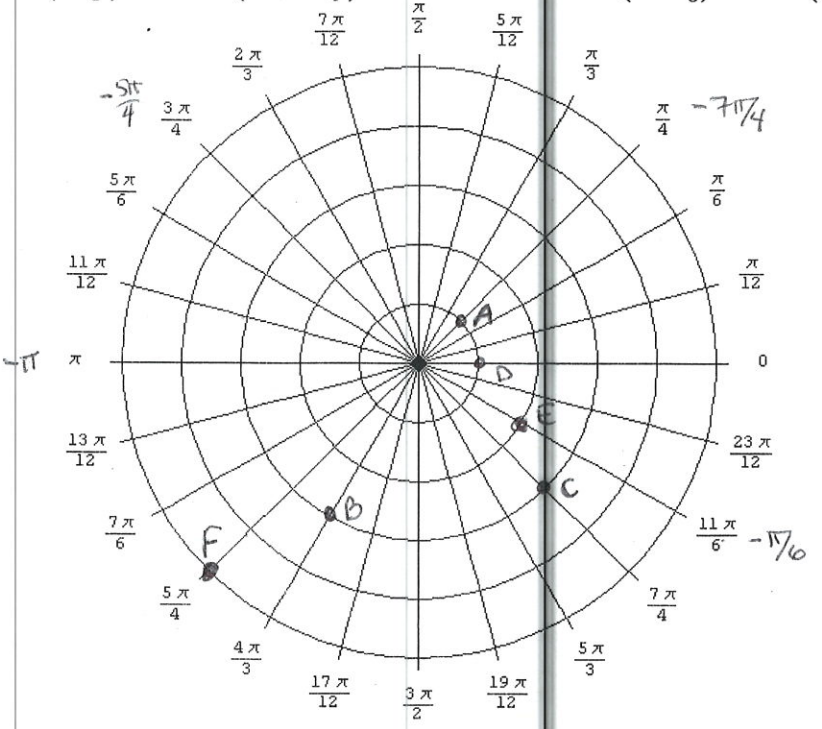
Instructions: Write your work up neatly and attach to this page. Record your final answers (only) directly on this page if they are short; if too long indicate which page of the work the answer is on and mark it clearly. Use exact values unless specifically asked to round.

1. Solve each triangle below. If using the law of sines, verify how many triangles are possible. If there are two possible triangles, find BOTH. Round to the nearest tenth.
   a. $A = 44°, B = 25°, a = 12$
   b. $A = 65°, B = 65°, c = 6$
   c. $a = 42.1, b = 37, A = 112°$
   d. $a = 7, b = 28, A = 12°$
   e. $a = 5, b = 7, C = 42°$
   f. $a = 63, b = 22, c = 50$
   g. $B = 85°, C = 15°, b = 40$
   h. $a = 30, b = 20, A = 50°$
   i. $a = 10, b = 30, A = 150°$
   j. $a = 9.3, b = 41, A = 18°$
   k. $a = 7, c = 3, B = 90°$
   l. $a = 5, b = 7, c = 10$

2. Two fire-lookout stations are 10 miles apart, with station B directly east of station A. Both stations spot a fire. The bearing from station A is N25°E and from station B it's N56°W. How far from each station is the fire (to the nearest tenth of a mile)?

3. You are on a fishing boat that leaves its pier and heads east. After traveling 30 miles, there is a report warning of rough seas directly south. The captain turns the boat and follows a bearing of S45°W for 12 miles. How far are you from the boat's pier? What bearing could the board have originally taken to arrive at this point?

4. Convert the points below into rectangular coordinates, and plot the points below on the graph. Label each.
   a. $\left(1, \frac{\pi}{4}\right)$ b. $\left(3, \frac{4\pi}{3}\right)$ c. $\left(-3, -\frac{5\pi}{4}\right)$ d. $(-1, -\pi)$ e. $\left(2, -\frac{\pi}{6}\right)$ f. $\left(-5, -\frac{7\pi}{4}\right)$



MTH
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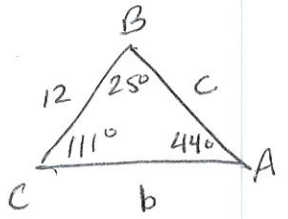
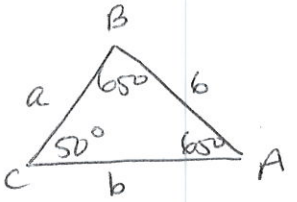
1.a. $A=44°$, $B=25°$, $a=12$



$$\frac{12}{\sin 44°} = \frac{b}{\sin 25°} \Rightarrow b = 7.3$$

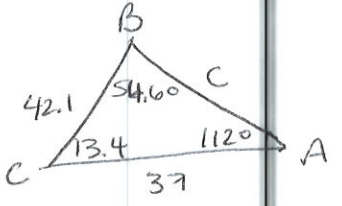
$$\frac{12}{\sin 44°} = \frac{c}{\sin 111°} \Rightarrow c = 16.1$$

b. $A=65°$, $B=65°$, $c=6$



$$\frac{6}{\sin(50°)} = \frac{a}{\sin(65°)} \quad a=b=7.1$$

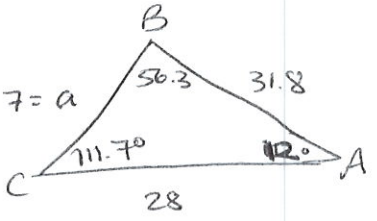
c. $a=42.1$, $b=37$, $A=112°$



$$\frac{\sin 112°}{42.1} = \frac{\sin B}{37} \quad \sin B = .81486 \Rightarrow B = 54.6°$$

$$\frac{\sin 112°}{42.1} = \frac{\sin 13.4°}{c} \quad c = 10.5$$

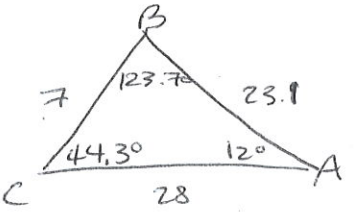
d. $a=7$, $b=28$, $A=12°$



$$\frac{\sin 12°}{7} = \frac{\sin B}{28}$$

$\sin B = .8316 \Rightarrow$ $B = 56.3°$, $C = 111.7°$

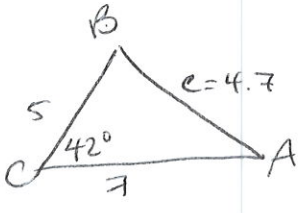
$$\frac{7}{\sin 12°} = \frac{c}{\sin 111.7°} \Rightarrow c = 31.3$$



$B = 123.7°$

$c = 23.1$

e. $a=5$, $b=7$, $C=42°$



$$c^2 = 7^2 + 5^2 - 2(5)(7)\cos 42°$$

$$c^2 = 21.98$$

$$c = 4.69 \Rightarrow 4.7$$

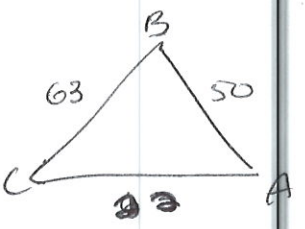
$$\cos B = \frac{7^2 - 5^2 - 4.7^2}{-2(5)(4.7)}$$

$B = 92.3°$

$$\cos A = \frac{5^2 - 7^2 - 4.7^2}{-2(7)(4.7)}$$

$A = 45.7°$

f. $a = 63, b = 22, c = 50$



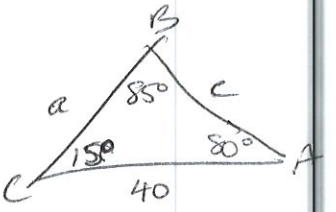
$$\cos C = \frac{50^2 - 63^2 - 22^2}{-2 \cdot 63 \cdot 22}$$

$C = 45.2°$

$$\cos B = \frac{22^2 - 63^2 - 50^2}{-2(63)(50)}$$

$B = 18.2°$ $\quad A = 116.6°$

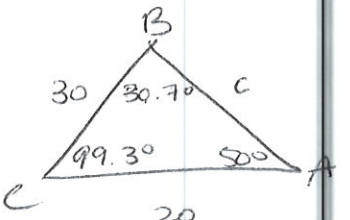
g. $B = 85°, C = 15°, b = 40$

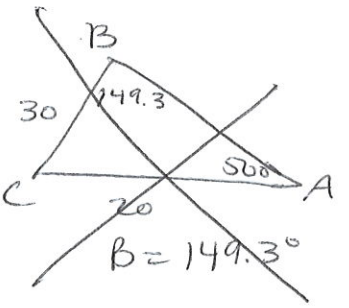


$$\frac{a}{\sin 80°} = \frac{40}{\sin 85°} \quad a = 39.5$$

$$\frac{c}{\sin 15°} = \frac{40}{\sin 85°} \quad c = 10.4$$

h. $a = 30, b = 20, A = 50°$



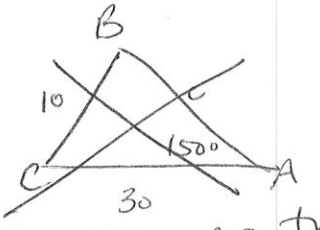


$$\frac{\sin 50°}{30} = \frac{\sin B}{20} \quad \sin B = .5107 \Rightarrow B = 30.7°.$$

$B = 149.3°$

$$\frac{\sin 50°}{30} = \frac{\sin 99.3}{c} \quad c = 38.6$$

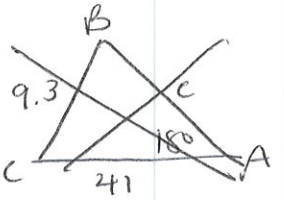
i. $a = 10, b = 30, A = 150°$



$$\frac{\sin 150°}{10} = \frac{\sin B}{30}$$

$\sin B = 1.5$ no triangle

j. $a = 9.3, b = 41, A = 18°$

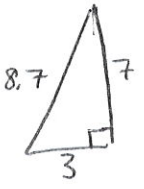


$$\frac{\sin 18°}{9.3} = \frac{\sin B}{41}$$

$\sin B = 1.36$

no triangle

k. $a = 7, c = 3, B = 90°$



$B^2 = 7^2 + 3^2$

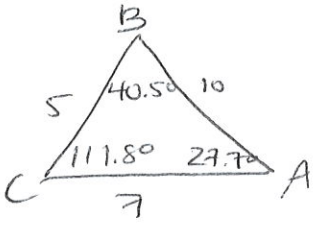
$A = 53.6°$

$C = 36.4°$

1d. $a=5, b=7, c=10$

$\cos C = \frac{10^2 - 5^2 - 7^2}{-2(5)(7)}$

$C = 111.8°$

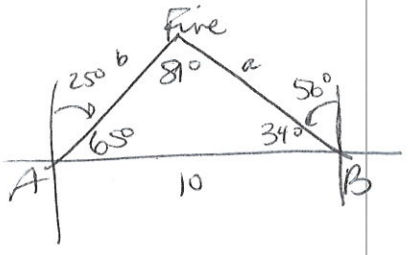


$\cos B = \frac{7^2 - 5^2 - 10^2}{-2(5)(10)}$

$B = 40.5°$

$A = 27.7°$

2.

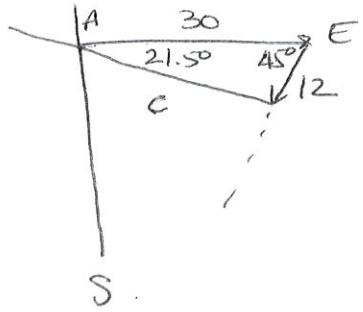


$\frac{10}{\sin 81°} = \frac{b}{\sin 34°}$ $\quad b = 5.7 \leftarrow$ dist. to A

$\frac{10}{\sin 81°} = \frac{a}{\sin 65°}$ $\quad a = 9.2$ – dist to B

Station A is closer

3.



$c^2 = 30^2 + 12^2 - 2(12)(30)\cos 45°$

$c = 23.1$

$\cos A = \frac{12^2 - 30^2 - 23.1^2}{-2(30)(23.1)} \Rightarrow A = 21.5°$

S 68.5° E

4. a. $(1, \pi/4)$ $\quad x = 1\cos \pi/4 = 1/\sqrt{2}$ $\quad y = 1\sin \pi/4 = 1/\sqrt{2}$ $\quad \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

b. $(3, 4\pi/3)$ $\quad x = 3\cos 4\pi/3 = -\frac{3}{2}$ $\quad y = 3\sin 4\pi/3 = -\frac{3\sqrt{3}}{2}$ $\quad \left(-\frac{3}{2}, -\frac{3\sqrt{3}}{2}\right)$

c. $(-3, -5\pi/4)$ $\quad x = -3\cos(-5\pi/4) = \frac{3}{\sqrt{2}}$ $\quad y = -3\sin(-5\pi/4) = -\frac{3}{\sqrt{2}}$ $\quad \left(\frac{3}{\sqrt{2}}, -\frac{3}{\sqrt{2}}\right)$

d. $(-1, -\pi)$ $\quad x = -1\cos(-\pi) = 1$ $\quad y = -1\sin(-\pi) = 0$ $\quad (1, 0)$

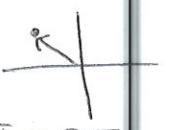
e. $(2, -\pi/6)$ $\quad x = 2\cos(-\pi/6) = \sqrt{3}$ $\quad y = 2\sin(-\pi/6) = -1$ $\quad (\sqrt{3}, -1)$

f. $(-5, -7\pi/4)$ $\quad x = -5\cos(-7\pi/4) = -5/\sqrt{2}$ $\quad y = -5\sin(-7\pi/4) = -5/\sqrt{2}$ $\quad \left(-\frac{5}{\sqrt{2}}, -\frac{5}{\sqrt{2}}\right)$

5. a. $(-2,2)$ $r^2 = x^2+y^2 \Rightarrow r=\sqrt{8}$

$4+4=8$

$\theta = \tan^{-1}\left(\frac{2}{-2}\right) = -\pi/4 + \pi = 3\pi/4$

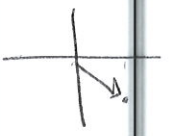


$(\sqrt{8}, 3\pi/4)$

b. $(2,-2\sqrt{3})$ $r^2 = 2^2 + (-2\sqrt{3})^2 =$

$4+12=16$

$r=4$



$(4, -\pi/3)$

$\theta = \tan^{-1}\left(\frac{-2\sqrt{3}}{2}\right) = \pi/3$

c. $(5,0)$ $r=5 \quad \theta = 0$ $(5,0)$

d. $(0,-6)$ $r=6, \ \theta = 3\pi/2$

$(6, 3\pi/2)$

e. $(-1,-\sqrt{3})$ $r^2 = (-1)^2 + (-\sqrt{3})^2 = 4$

$r=2$

$\theta = \tan^{-1}\left(\frac{-\sqrt{3}}{-1}\right) = \pi/3 + \pi = 4\pi/3$

$(2, 4\pi/3)$

6. $3x+y=7$

a. $3r\cos\theta + r\sin\theta = 7 \rightarrow r(3\cos\theta + \sin\theta) = 7$

$r = \dfrac{7}{3\cos\theta + \sin\theta}$

b. $x^2+y^2=9 \Rightarrow r^2=9 \Rightarrow r=3$

c. $y^2 = 6x \Rightarrow r^2\sin^2\theta = 6r\cos\theta$

$r = \dfrac{6\cos\theta}{\sin^2\theta} \Rightarrow r = 6\cot\theta\csc\theta$

d. $y=3 \Rightarrow r\sin\theta = 3 \Rightarrow r = 3\csc\theta$

e. $x^2+(y+3)^2 = 9 \Rightarrow x^2+y^2+6y+9 = 9 \Rightarrow x^2+y^2 = -6y$

$r^2 = -6r\sin\theta \Rightarrow r = -6\sin\theta$

7a. $r=8 \Rightarrow r^2=64 \Rightarrow x^2+y^2=64$

b. $\theta = \frac{2\pi}{3} \Rightarrow \tan\theta = \frac{y}{x} = -\sqrt{3} \Rightarrow y = -\sqrt{3}x$

c. $r\sin\theta = 3 \Rightarrow y=3$

d. $r = 8\cos\theta + 2\sin\theta \Rightarrow r^2 = 8r\cos\theta + 2r\sin\theta$

$x^2+y^2 = 8x+2y$

5. Convert the following points to polar coordinates.

a. $(-2,2)$ b. $(2,-2\sqrt{3})$ c. $(5,0)$ d. $(0,-6)$ e. $(-1,-\sqrt{3})$

6. Convert the equation to polar coordinates. Solve for r when feasible.

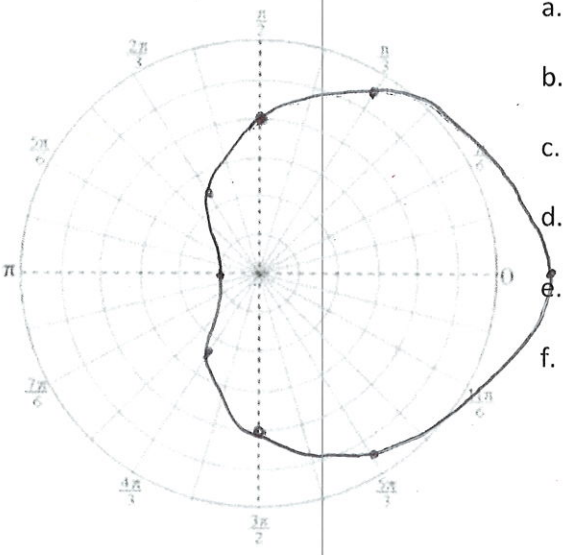
a. $3x + y = 7$
b. $x^2 + y^2 = 9$
c. $y^2 = 6x$
d. $y = 3$
e. $x^2 + (y+3)^2 = 9$

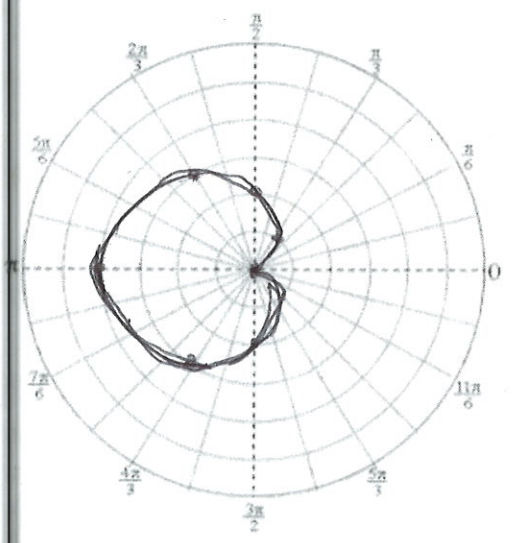
7. Convert the polar coordinates to rectangular coordinates.

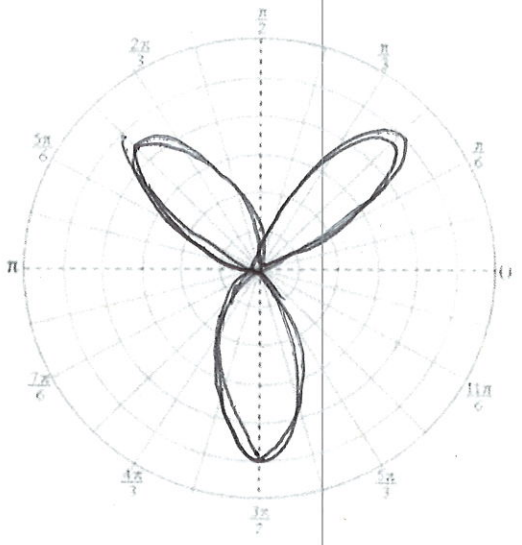
a. $r = 8$
b. $\theta = \frac{2\pi}{3}$
c. $r\sin\theta = 3$
d. $r = 8\cos\theta + 2\sin\theta$
e. $r^2\sin 2\theta = 4$
f. $r = 12\cos\theta$

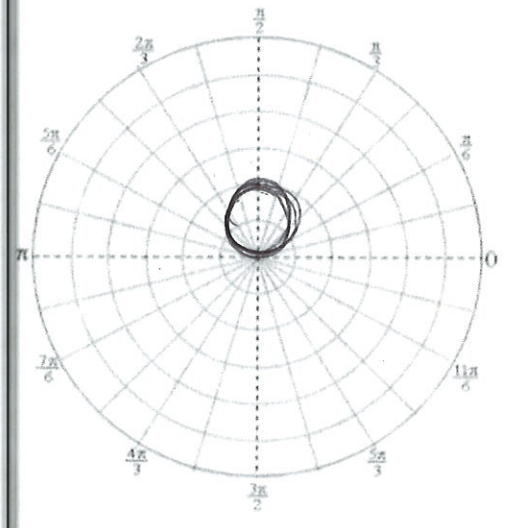
8. Graph the following functions in polar coordinates by hand on the graphs below.

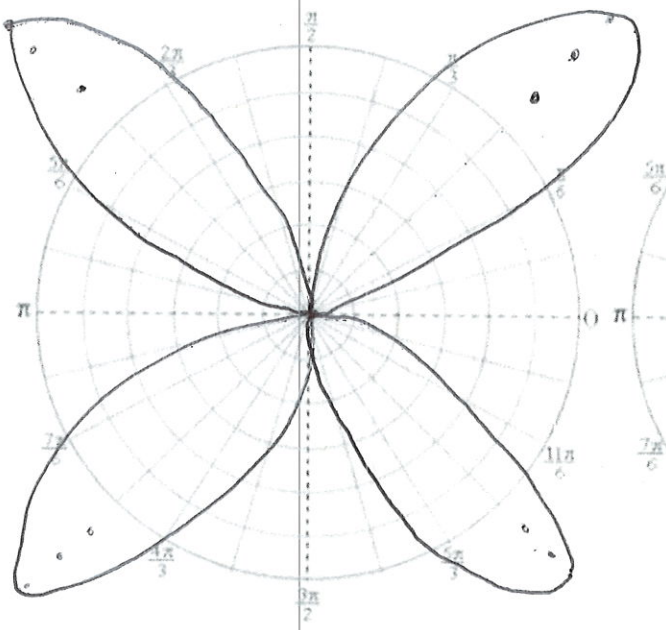
a. $r = 4 + 3\cos\theta$
b. $r = 2 - 2\cos\theta$
c. $r = 4\sin 3\theta$
d. $r = 2\sin\theta$
e. $r^2 = 9\sin 2\theta$
f. $r\cos\theta = -3$

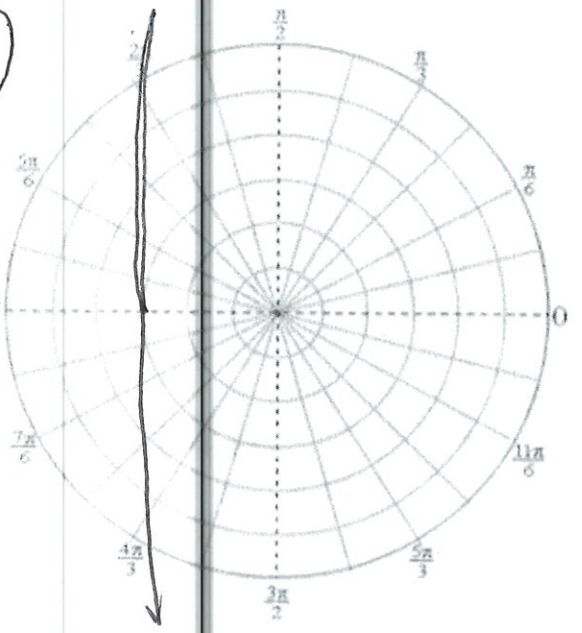












9. Use technology to graph the equations below. Sketch the results. For some graphs $[0,4\pi]$ should be fine, but for others, you may want $[-2\pi, 2\pi]$ as the domain for θ.

a. $r = \cos(\frac{3}{2}\theta)$

b. $r = \frac{1}{1-sin\theta}$

c. $r = \cos^2 5\theta + \sin 3\theta + 0.3$

d. $r = \sin^4 \theta + \cos 3\theta$

e. $r = 1.5^{\sin\theta}$

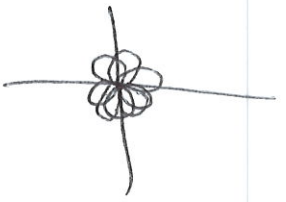
f. $r = \ln\left(\theta^2 + \frac{\pi}{4}\right)$

7e. $r^2 \sin 2\theta = 4 \Rightarrow r^2(2\sin\theta\cos\theta) = 4 \Rightarrow$

$r\sin\theta \, r\cos\theta = 2 \Rightarrow xy = 2 \Rightarrow y = \frac{2}{x}$

f. $r = 12\cos\theta \Rightarrow r^2 = 12r\cos\theta \Rightarrow x^2 + y^2 = 12x$

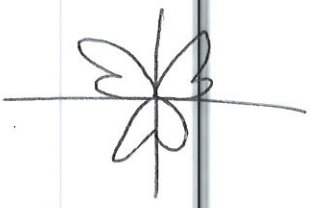
9. a. $r = \cos\left(\frac{3}{2}\theta\right)$



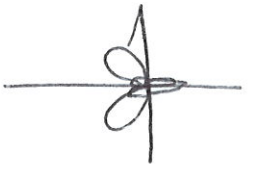
b. $r = \frac{1}{1-\sin\theta}$

c. $r = \cos^2 5\theta + \sin 3\theta + .3$

d. $r = \sin^4\theta + \cos 3\theta$



e. $r = 1.5^{\sin\theta}$



f. $r = \ln(\theta^2 + \pi/4)$