MAT 161, Practice Final Exam, Fall 2018 Name KEY

Instructions: Show all work. Give exact answers unless specifically asked to round. Complete all parts of each question. Questions that provide only answers and no work will not receive full credit. If you use your calculator (only when problems don't instruct you to do the problem by hand), showing calculator steps will count as "work".

1. Solve the system $\begin{cases} 5x + 12y + z = 10 \\ 2x + 5y + 2z = -1 \\ x + 2y - 3z = 5 \end{cases}$ by any method. (12 points)

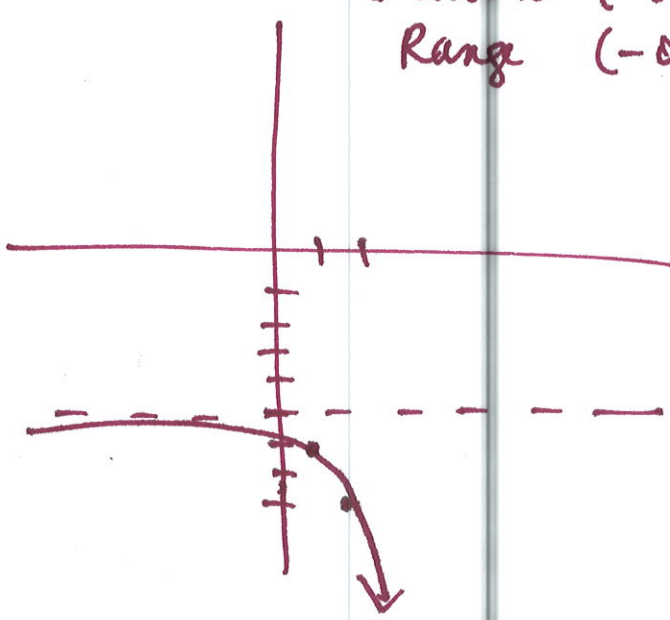
$$\left[\begin{array}{ccc|c} 5 & 12 & 1 & 10 \\ 2 & 5 & 2 & -1 \\ 1 & 2 & -3 & 5 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & -19 & 0 \\ 0 & 1 & 8 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

$0 = 1$ no solution

inconsistent

2. Sketch the graph of the curve $f(x) = -3^{x-1} - 5$. State the domain and range. (10 points)

Domain $(-\infty, \infty)$
Range $(-\infty, -5)$



3. Find the inverse function of $f(x) = 10e^x + 7$. State the domain and range of the inverse. (8 points)

$$x = 10e^y + 7$$
$$x - 7 = 10e^y$$
$$\frac{x-7}{10} = e^y$$
$$\ln\left(\frac{x-7}{10}\right) = y$$

Inverse
Domain $(7, \infty)$
Range $(-\infty, \infty)$

4. Solve the following equations without using a calculator. (8 points each)

a. $\log(x + 4) - \log 2 = \log(5x + 1)$

$$\cancel{\log}[(x+4)/2] = \cancel{\log}(5x+1)$$
$$\frac{x+4}{2} = 5x+1$$
$$x + 4 = 10x + 2 \rightarrow 2 = 9x$$
$$\boxed{x = 9/2}$$

b. $e^{2x} - 4e^x - 12 = 0$

$u = e^x$

$$u^2 - 4u - 12 = 0$$
$$(u-6)(u+2) = 0$$
$$u = 6 \quad u = -2$$

$u = e^x = 6$

$\boxed{x = \ln 6}$

$u = e^x = -2$ no solution

5. Find $\frac{f(x+h)-f(x)}{h}$ for $f(x) = -2x^2 + 3x - 5$. (10 points)

$$\frac{-2(x+h)^2 + 3(x+h) - 5 - (-2x^2 + 3x - 5)}{h} =$$

$$\frac{-2(x^2 + 2xh + h^2) + 3x + 3h - 5 + 2x^2 - 3x + 5}{h} =$$

$$\frac{-\cancel{2x^2} - 4xh - 2h^2 + \cancel{3x} + 3h - \cancel{5} + \cancel{2x^2} - \cancel{3x} + \cancel{5}}{h} =$$

$$\frac{-4xh - 2h^2 + 3h}{h} = \frac{\cancel{h}(-4x - 2h + 3)}{\cancel{h}} = \boxed{-4x - 2h + 3}$$

6. If $f(x) = |x|$, write the function that has all the following transformations applied: (8 points)
   a. Shift left 4 units
   b. Reflect over the x-axis
   c. Compress by a factor of 2
   d. Shift up by 3

$g(x) = -\frac{1}{2}|x+4|+3$

7. Given $g(x) = \sqrt{x-4}$, $h(x) = x + \frac{1}{x}$, find $(h \circ g)(x)$ and state the domain. (8 points)

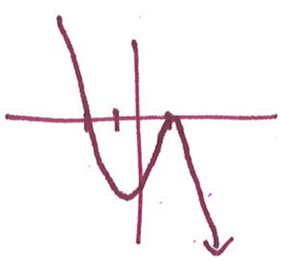
$h(g(x))$

$h \circ g = \sqrt{x-4} + \frac{1}{\sqrt{x-4}}$

Domain: $x > 4$ or $(4, \infty)$

8. Find all the possible rational zeros of the polynomial $f(x) = -x^3 - x^2 + 5x - 3$. Use them to factor the polynomial, and find all the real (and complex, if any) zeros. Write the polynomial in factored form. (10 points)

Zeros at $x = -3$, $x = 1$



$$
\begin{array}{r}
-x^2 + 2x - 1 \\
x+3 \overline{) -\cancel{x^3} - x^2 + 5x - 3} \\
+\cancel{x^3} + 3x^2 \\
\hline
2\cancel{x^2} + 5x \\
-2\cancel{x^2} \mp 6x \\
\hline
-x - 3 \\
+x + 3 \\
\hline
0
\end{array}
$$

$-(x+3)(x^2-2x+1)$

$= -(x+3)(x-1)^2$

9. Sketch the graph of the function $f(x) = \frac{x^3+x}{x^2-4}$, but finding i) any intercepts, ii) any vertical asymptotes or holes, iii) any horizontal or slant asymptotes. (10 points)

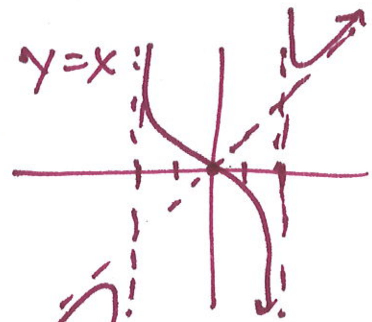
$x^2-4=0$ $\quad (x-2)(x+2)=0$ $\quad x=2,-2 \leftarrow$ VA

$x^3+x=0$ $\quad x(x^2+1)=0$ $\quad x=0$ intercept (y) & x

$$\begin{array}{r|l} & \quad x \\ x^2-4 & \cancel{x^3}+0x^2+x+0 \\ & -\cancel{x^3} \qquad +4x \\ \hline & \qquad 5x \end{array}$$

$\rightarrow \quad x + \frac{5x}{x^2-4}$

slant asymptote $y=x$



10. Solve the rational and polynomial inequalities and write the solution in interval notation. (4 points each)

a. $x^3 - x^2 + 9x - 9 > 0$

$x^2(x-1)+9(x-1)>0$

$(x^2+9)(x-1)>0$

$\uparrow$

never 0 $\quad x=1$

$-$ $\quad$ $+$

1

$(1, \infty)$

b. $\frac{(x+3)(x-2)}{x+1} \leq 0$

$-$ $\quad$ $+$ $\quad$ $-$ $\quad$ $+$

$\underline{-3}$ $\quad$ -1 $\quad$ 0 $\quad$ 1 $\quad$ $\underline{2}$

$\neq$

$x=-4$ $\quad \frac{(-)(-)}{(-)}$

$x=-2$ $\quad \frac{(+)(-)}{(-)}$

$x=0$ $\quad \frac{(+)(-)}{(+)}$

$x=3$ $\quad \frac{(+)(+)}{(+)}$

$(-\infty, -3] \cup (-1, 2]$

11. Find the domain and range of the graph $f(x) = \log(x-2) - 1$. Sketch the graph. (6 points)

Domain $(2, \infty)$
Range $(-\infty, \infty)$

12. Expand the logarithmic function $\log\left[\frac{10x^2\sqrt[3]{1-x}}{7(x+1)^2}\right]$ into simpler logs and simplify expressions where possible. (5 points)

$$\log(10x^2\sqrt[3]{1-x}) - \log[(7)(x+1)^2] =$$

$$\log 10 + \log x^2 + \log\sqrt[3]{1-x} - [\log 7 + \log(x+1)^2] =$$

$$\log 10 - 2\log x + \tfrac{1}{3}\log(1-x) - \log 7 - 2\log(x+1)$$

13. Write the expanded logarithm $\frac{2}{3}[2\ln(x+5) - \ln x - \ln(x^2-4)]$ as a single logarithmic expression. (5 points)

$$\tfrac{2}{3}[\ln(x+5)^2 - \ln x - \ln(x^2-4)] =$$

$$\tfrac{2}{3}\left[\ln\left[\frac{(x+5)^2}{x(x^2-4)}\right]\right] = \ln\left(\frac{(x+5)^2}{x(x^2-4)}\right)^{2/3}$$

14. Solve the following equations without using a calculator. (5 points each)

a. $9^x = \frac{1}{\sqrt[3]{3}}$

$3^{2x} = 3^{-1/3}$

$2x = -\frac{1}{3} \rightarrow x = -\frac{1}{6}$

b. $e^{4x} - 3e^{2x} - 18 = 0$ $\quad u = e^{2x}$

$u^2 - 3u - 18 = 0$

$(u-6)(u+3) = 0$

$u = 6 \quad u = -3$

$e^{2x} = 6$

$\ln 6 = 2x$

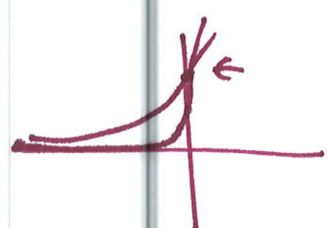
$\boxed{\frac{\ln 6}{2} = x}$

$e^{2x} = -3$ no sol.

15. Solve the following equations. You may use a calculator (show any graphs used). Round answers to 4 decimal places. (5 points)

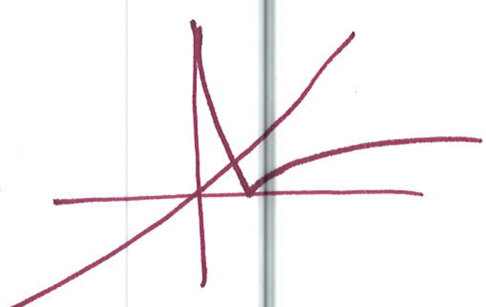
a. $7^{2x+1} = 3^{x+2}$

$X = 0.0899734$



b. $2|\ln x| = 2^{x-1} + x$

$X = 0.53338082$



16. Solve the systems below by the method of your choice, and characterize each as consistent or inconsistent, and if applicable, dependent or independent. (7 points each)

a. $\begin{cases} 3x - 2y = -5 \\ 4x + y = 8 \end{cases}$

$$\left[\begin{array}{cc|c} 3 & -2 & -5 \\ 4 & 1 & 8 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 4 \end{array}\right]$$

$(1, 4)$

b. $\begin{cases} x + 3y + 5z = 20 \\ y - 4z = -16 \\ 3x - 2y + 9z = 36 \end{cases}$

$$\left[\begin{array}{ccc|c} 1 & 3 & 5 & 20 \\ 0 & 1 & -4 & -16 \\ 3 & -2 & 9 & 36 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

$x = 0, y = 0, z = 4$

17. Rewrite $\frac{x^4+3x^2+x}{(x-1)(x+2)^2(x^2+4)}$ with partial fractions. Do not solve for the constants. (5 points)

$$= \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{(x+2)^2} + \frac{Dx+E}{x^2+4}$$