MTH 265, Quiz #13, Fall 2018   Name _____ KEY _____

Instructions: Show all work. Use exact answers unless otherwise asked to round.

1. Consider the function $f(x, y) = x^2 y - e^{x-y}$.
   a. Find the equation of the tangent plane at the point (1,1).

$$F = x^2 y - e^{x-y} - z$$

$$\nabla F = \langle 2xy - e^{x-y}, x^2 + e^{x-y}, -1 \rangle \qquad \nabla F(1,1) = \langle 1, 2, -1 \rangle$$

$$f(x,y) = 1 - 1 = 0 \qquad (1,1,0)$$

$$1(x-1) + 2(y-1) - 1(z - 0) = 0$$

   b. Find the equation of the normal line in vector-valued function form at the same point.

$$\vec{n}(t) = (t+1)\hat{\imath} + (2t+1)\hat{\jmath} + (-t)\hat{k}$$

2. Find the equation of the tangent plane for the parametric surface $\vec{r}(u, v) = u \cos v\, \hat{\imath} + (u \sin v - 1)\hat{\jmath} + u^2 \hat{k}$ at $u = 2\sqrt{2}, v = \frac{\pi}{4}$. $(2, 1, 8)$

$$\vec{r}_u = \cos v\, \hat{\imath} + \sin v\, \hat{\jmath} + 2u\hat{k}$$
$$\vec{r}_v = -u\sin v\, \hat{\imath} + u\cos v\, \hat{\jmath} + 0\hat{k}$$

$$\vec{r}_u \times \vec{r}_v = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \cos v & \sin v & 2u \\ -u\sin v & u\cos v & 0 \end{vmatrix} = (0 - 2u^2\cos v)\hat{\imath} - (0 + 2u^2\sin v)\hat{\jmath} + (u\cos^2 v + u\sin^2 v)\hat{k}$$

$$= -2u^2\cos v\,\hat{\imath} - 2u^2\sin v\,\hat{\jmath} + u\hat{k}$$

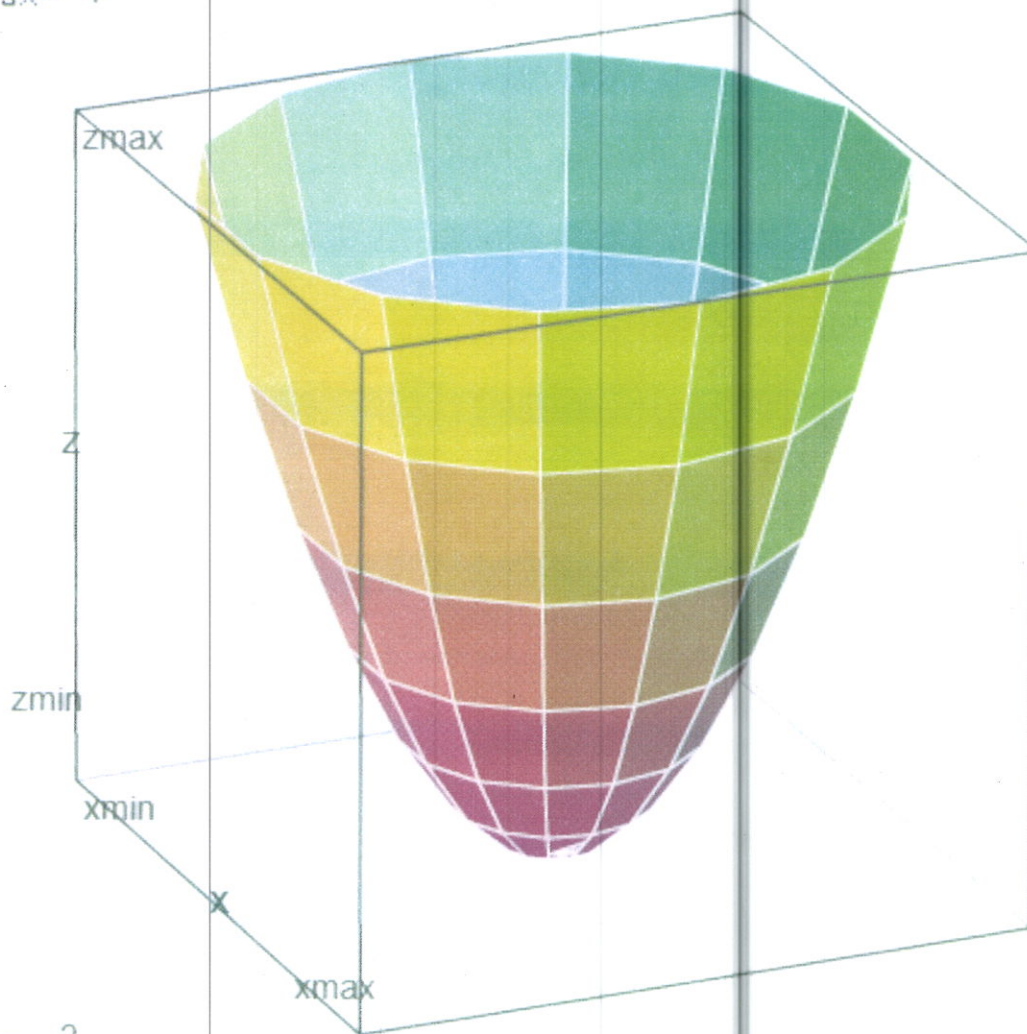$$\langle -8\sqrt{2}, -8\sqrt{2}, 2\sqrt{2} \rangle$$

$$-8\sqrt{2}(x-2) - 8\sqrt{2}(y-1) + 2\sqrt{2}(z-8) = 0$$

3. What kind of surface is the function in #2? Use technology to produce a graph.

paraboloid

zmin= 0

zmax= 4

zmax

z

zmin

xmin

x

xmax

xmin= -2

xmax= 2

Grid: 14 by 14

ymin= -3

ymax= 1