MTH 265, Quiz #15, Fall 2018 Name KEY

Instructions: Show all work. Use exact answers unless otherwise asked to round.

1. Find the surface area of the function $z = xy$ over the region bounded inside the cylinder $x^2 + y^2 = 2$.

$f = xy - z \quad \nabla f = \langle y, x, -1 \rangle \quad \iint_R \sqrt{y^2+x^2+1}\, dA$

$\int_0^{2\pi}\int_0^{\sqrt{2}} \sqrt{r^2+1}\, r\,dr\,d\theta = \int_0^{2\pi} \frac{1}{3}(r^2+1)^{3/2}\Big|_0^{\sqrt{2}} d\theta$

$u = r^2+1$
$du = 2r\,dr$
$\frac{1}{2}du = r\,dr$
$\frac{1}{2}\int u^{1/2}du = \frac{1}{2}\cdot\frac{2}{3}u^{3/2}$

$= \int_0^{2\pi} \frac{1}{3}[3^{3/2} - 1]\, d\theta = \frac{2\pi}{3}[3^{3/2} - 1]$

2. Set up the integral needed to find the surface area of the function $\vec{r}(u, v) = u^2 \cos v\, \hat{\imath} + u^2 \sin v\, \hat{\jmath} + uv\hat{k}$ over the region $0 \le u \le 3, 0 \le v \le 2\pi$. You do not need to integrate.

$r_u = 2u\cos v\,\hat{\imath} + 2u\sin v\,\hat{\jmath} + v\hat{k}$

$r_v = -u^2\sin v\,\hat{\imath} + u^2\cos v\,\hat{\jmath} + uk$

$\iint_R \|r_u \times r_v\|\, dA$

$= \int_0^{2\pi}\int_0^3 u\sqrt{(2\sin v - v\cos v)^2 + (2\cos v - v\sin v)^2 + 4u^2}\, du\,dv$

$\vec{r}_u \times \vec{r}_v = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2u\cos v & 2u\sin v & v \\ -u^2\sin v & u^2\cos v & u \end{vmatrix} = (2u^2\sin v - u^2 v\cos v)\hat{\imath} - (2u^2\cos v + u^2 v\sin v)\hat{\jmath} + (2u^3\cos^2 v + 2u^3\sin^2 v)\hat{k}$

$= 2u^3\hat{k}$

3. Use the Fundamental Theorem of Line Integrals to evaluate $\int_C \vec{F}\cdot d\vec{r}$ for the vector field $\vec{F}(x, y, z) = yze^{xz}\hat{\imath} + e^{xz}\hat{\jmath} + xye^{xz}\hat{k}$ on the curve $C: \vec{r}(t) = (t^2 + 1)\hat{\imath} + (t^2 - 1)\hat{\jmath} + (t^2 - 2)\hat{k}$, $0 \le t \le 2$.

$r(0) = 1\hat{\imath} - 1\hat{\jmath}, -2\hat{k}$
$r(2) = 5\hat{\imath} + 3\hat{\jmath} + 2\hat{k}$

$\int yze^{xz}dx = ye^{xz} + f(y,z)$

$\int e^{xz}dy = ye^{xz} + g(x,z)$

$\int xye^{xz}dz = ye^{xz} + h(x,y) \qquad \varphi = ye^{xz}$

$\int_C F\cdot dr = \varphi(5,3,2) - \varphi(1,-1,-2)$

$3e^{10} - (-1)e^{-2} = 3e^{10} + \frac{1}{e^2}$

4. Use Green's Theorem to evaluate $\int_C xy^2dx + 2x^2ydy$ where C is the boundary of the region $y = x^2, y = x$.

M N

$\frac{\partial N}{\partial x} = 4xy \qquad \frac{\partial M}{\partial y} = 2xy$

$\int_0^1\int_{x^2}^{x} (4xy - 2xy)\, dy\,dx = \int_0^1\int_{x^2}^{x} 2xy\, dy\,dx = \int_0^1 xy^2\Big|_{x^2}^{x} dx =$

$\int_0^1 x^3 - x^5\, dx = \frac{1}{4}x^4 - \frac{1}{6}x^6\Big|_0^1 = \frac{1}{12}$