MTH 265, Quiz #6, Fall 2018 Name KEY

Instructions: Show all work. Use exact answers unless otherwise asked to round.

1. Find ∇f and $\nabla^2 f$ for the function $f(x,y,z) = \frac{1}{2}xy^2\cos(y+z^3)$.

$$\nabla f = \langle \tfrac{1}{2}y^2\cos(y+z^3),\ xy\cos(y+z^3) - \tfrac{1}{2}xy^2\sin(y+z^3),\ -\tfrac{1}{2}xy^2\sin(y+z^3)3z^2 \rangle$$

$$\nabla^2 f = 0 + x\cos(y+z^3) - xy\sin(y+z^3) - xy\sin(y+z^3) - \tfrac{1}{2}xy^2\cos(y+z^3) - 3xy^2z\sin(y+z^3) - \tfrac{3}{2}xy^2z^2\cos(y+z^3)\cdot 3z^2$$

2. Find $\nabla \cdot \vec{F}$ and $\nabla \times \vec{F}$ for $\vec{F}(x,y,z) = \sin(xy)\,\hat{\imath} - \cos(yz)\,\hat{\jmath} + \tan(xz)\hat{k}$.

$$\vec{\nabla}\cdot\vec{F} = y\cos(xy) + z\sin(yz) + x\sec^2(xz)$$

$$\vec{\nabla}\times\vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ \sin xy & -\cos yz & \tan xz \end{vmatrix} = (0 - y\sin yz)\hat{\imath} - (z\sec^2 xz - 0)\hat{\jmath} + (0 - x\cos xy)\hat{k}$$

$$= -y\sin yz\,\hat{\imath} - z\sec^2 xz\,\hat{\jmath} - x\cos xy\,\hat{k}$$

3. Determine if the vector field $\vec{F}(x,y,z) = (x+y)\hat{\imath} + (y-z)\hat{\jmath} + z^2\hat{k}$ is conservative.

$$\vec{\nabla}\times\vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x+y & y-z & z^2 \end{vmatrix} = (0+1)\hat{\imath} - (0-0)\hat{\jmath} + (0-1)\hat{k}$$

$$\langle 1, 0, -1 \rangle$$

not conservative