MTH 266, Exam #2, Part I, Fall 2018 Name ____

Instructions: Show all work. Give exact answers unless specifically asked to round. If you do not show work, problems will be graded as “all or nothing” for the answer only; partial credit will not be possible and any credit awarded for the work will not be available. On this portion of the exam, you may **NOT** use a calculator.

1. Compute the determinant by the cofactor method. (15 points)

$$\begin{vmatrix} 0 & 3 & 9 & 5 \\ 1 & 0 & -2 & 4 \\ -3 & 2 & 1 & 6 \\ 0 & 8 & 2 & 3 \end{vmatrix}$$

2. Compute the determinant by using row operations. (12 points)

$$\begin{vmatrix} 1 & -1 & 5 \\ 2 & 4 & -3 \\ 3 & -2 & 0 \end{vmatrix}$$

3. Determine if the following sets are linearly independent or dependent. Justify your answers **without performing matrix calculations.** (5 points each)

a. $\left\{ \begin{bmatrix} 1 \\ 7 \\ -3 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 6 \\ 5 \\ 1 \end{bmatrix}, \begin{bmatrix} 6 \\ 11 \\ 17 \end{bmatrix} \right\}$

b. $\left\{ \begin{bmatrix} 1 \\ 7 \\ -3 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix} \right\}$

4. Given that A and B are $n \times n$ matrices with det $A = 3$ and det $B = -4$, find the following. (5 points each)

a) $det\ AB$

d) $det\ A^T$

b) $det\ A^{-1}$

e) $det\ 2A$

c) $det\ (-AB^2)$

MTH 266, Exam #2, Part II, Fall 2018 Name ______________________________________

Instructions: Show all work. Give exact answers unless specifically asked to round. All complex numbers should be stated in standard form, and all complex fractions should be simplified. If you do not show work, problems will be graded as "all or nothing" for the answer only; partial credit will not be possible and any credit awarded for the work will not be available. On this portion of the exam, you *may* use a calculator to perform elementary matrix operations. Support your answers with work (reproduce the reduced matrices from your calculator) or other justification for full credit.

1. Determine if each statement is True or False. (2 points each)

a. T F If matrix B is formed by multiplying a row of matrix A by 4, then $det\ B\ =\ 4\ det\ A$

b. T F The equation $A\mathbf{x} = \mathbf{0}$ has only the trivial solution if and only if there are no free variables.

c. T F If an $m \times n$ matrix has a pivot in every row, then the equation $A\mathbf{x} = \mathbf{b}$ has a unique solution for each **b** in R^m.

d. T F If $\{\mathbf{u},\ \mathbf{v},\ \mathbf{w}\}$ is linearly independent, then **u**, **v**, and **w** are not in R^2.

e. T F If *A* and *B* are $m \times n$ matrices, then both AB^{T} and $A^{\mathrm{T}}B$ are defined.

f. T F If two rows of a 3×3 matrix *A* are the same, then det *A* = 0.

g. T F If $\{\mathbf{v}_1, \ldots, \mathbf{v}_{\mathrm{p}}\}$ is linearly independent, then so is $\{\mathbf{v}_1, \ldots, \mathbf{v}_{\mathrm{p}-1}\}$.

h. T F The pivot columns of a matrix are always linearly dependent.

i. T F The rank of a matrix is defined by the dimension of the null space.

j. T F If det A is zero, then two rows or two columns of A are the same, or a row or a column is zero.

k. T F If A and B are row equivalent, then their column spaces are the same.

l. T F The vector space $\mathbb{P}_3$ and R^3 are isomorphic.

m. T F A linearly independent set in a subspace H is a basis for H.

n. T F If P_B is the change-of-coordinates matrix, then $\left[\vec{x}\right]_B = P_B\vec{x}$ for $\vec{x}$ in V.

2. Determine if the columns of $A = \begin{bmatrix} 2 & 5 & 2 \\ 0 & 4 & -1 \\ 3 & -2 & 1 \\ 0 & 1 & -1 \end{bmatrix}$ form a linearly independent set and justify your answer. (10 points)

3. Given $\mathrm{T} : \mathrm{R}^3 \to \mathrm{R}^3$ such that $T(\mathbf{x}) = \begin{bmatrix} 3x_1 - 2x_2 + x_3 \\ x_2 + 4x_3 \\ -2x_1 + 3x_3 \end{bmatrix}$ answer the following.

   a. Find the standard matrix, A, such that $T(\mathbf{x}) = A\mathbf{x}$. (7 points)

   b. Is *T* onto R^3? Justify your answer. (5 points)

   c. Is T one-to-one? Justify your answer. (5 points)

4. Determine if the set $H = \left\{ \begin{bmatrix} 1 \\ 7 \\ -3 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 6 \\ 5 \\ 1 \end{bmatrix} \right\}$ forms a basis for R^3. Justify your answer.

(10 points)

5. Assume that $A = \begin{bmatrix} 1 & 2 & -5 & 11 & -3 \\ 2 & 4 & -5 & 15 & 2 \\ 1 & 2 & 0 & 4 & 5 \\ 3 & 6 & -5 & 19 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 0 & 4 & 5 \\ 0 & 0 & 5 & -7 & 8 \\ 0 & 0 & 0 & 0 & -9 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ are row equivalent.

   a. Find a basis for the column space of A and state the dimension of $Col\ A$. (8 points)

   b. Find a basis for the null space of A and state the dimension of $Nul\ A$. (8 points)

c. Determine if $\mathbf{b} = \begin{bmatrix} -2 \\ 0 \\ 4 \\ -3 \end{bmatrix}$ is in $Col\ A$. Show appropriate work to justify your answer.

(8 points)

d. What is the rank of A? (3 points)

6. Suppose matrix A is a 6×8 matrix with 5 pivot columns. Determine the following. (18 points)

$dim\ Col\ A$ = _______ $dim\ Nul\ A$ = _______

$dim\ Row\ A$ = _______ If $Col\ A$ is a subspace of R^m, then m= ________

$Rank\ A$ = _______ If $Nul\ A$ is a subspace of R^n, then n = ________

7. Given the bases $B = \{b_1, b_2, b_3\}$ and $C = \{c_1, c_2, c_3\}$ below, find the change of basis matrices $\underset{C\leftarrow B}{P}$ and $\underset{B\leftarrow C}{P}$. If the B-coordinate vector for $\vec{x}$ is as shown, find the C-coordinate vector for $\vec{x}$. (20 points)

$$\vec{b_1} = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}, \vec{b_2} = \begin{bmatrix} 2 \\ 0 \\ 8 \end{bmatrix}, \vec{b_3} = \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}, \vec{c_1} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}, \vec{c_2} = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}, \vec{c_3} = \begin{bmatrix} 0 \\ -1 \\ -2 \end{bmatrix}, \left[\vec{x}\right]_B = \begin{bmatrix} 1 \\ 0 \\ -9 \end{bmatrix}$$

8. Given the basis $B = \{1-t^2, t-t^2, 2-t+t^2, 2t-t^2+t^3\}$ for $\mathbb{P}_3$. Find $\vec{p}(t) = 2+5t-7t^3$ in this basis. (12 points)

9. Prove that $\det A^{-1} = \dfrac{1}{\det A}$ assuming that A is invertible. [Hint: use multiplication properties of the determinant and what you know about $n \times n$ identity matrices.] (8 points)

10. List at least 8 properties of Invertible Matrices from the Invertible Matrix Theorem. (8 points)