MTH 266, Quiz #11, Fall 2018 Name KEY

Instructions: Show all work. Use exact answers unless otherwise asked to round.

1. Given $\vec{u}_1 = \begin{bmatrix} 5 \\ -4 \\ 0 \\ 3 \end{bmatrix}, \vec{u}_2 = \begin{bmatrix} 4 \\ 5 \\ 1 \\ 0 \end{bmatrix}$ and $W = Span\{\mathbf{u}_1, \mathbf{u}_2\}$. Determine if $\{\mathbf{u}_1, \mathbf{u}_2\}$ is an orthogonal basis for W. If it is, make it an orthonormal basis.

$u_1 \cdot u_2 = 5(4) - 4(5) + 0(1) + 3(0) = 0$

they are orthogonal

$\|\hat{u}_1\| = \sqrt{25 + 16 + 9} = \sqrt{50} = 5\sqrt{2}$

$\|\hat{u}_2\| = \sqrt{16 + 25 + 1} = \sqrt{42}$

$\hat{u}_1 = \begin{bmatrix} 5/5\sqrt{2} \\ -4/5\sqrt{2} \\ 0 \\ 3/5\sqrt{2} \end{bmatrix}$ $\hat{u}_2 = \begin{bmatrix} 4/\sqrt{42} \\ 5/\sqrt{42} \\ 1/\sqrt{42} \\ 0 \end{bmatrix}$ forms an orthonormal basis

2. Given the basis of W in question #1, and the vector $\vec{y} = \begin{bmatrix} 5 \\ 2 \\ 1 \\ -1 \end{bmatrix}$ decompose this vector into $\vec{y} = \vec{y_\parallel} + \vec{y_\perp}$ with $\vec{y_\parallel} = proj_W \vec{y}$.

$\vec{u}_1 \cdot \vec{y} = 5(5) - 4(2) + 0(1) + 3(-1) = 25 - 8 - 3 = 14$

$\|u_1\|^2 = 50$

$proj_{\hat{u}_1} y = \frac{14}{50} \begin{bmatrix} 5 \\ -4 \\ 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 7/5 \\ -28/25 \\ 0 \\ 21/25 \end{bmatrix}$

$\bar{u}_2 \cdot y = 4(5) + 5(2) + 1(1) + 0(-1) = 20 + 10 + 1 = 31$

$\|u_2\|^2 = 42$

$proj_{\hat{u}_2} y = \frac{31}{42} \begin{bmatrix} 4 \\ 5 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 62/21 \\ 155/42 \\ 31/42 \\ 0 \end{bmatrix}$

$\vec{y}_\parallel = \begin{bmatrix} 7/5 \\ -28/25 \\ 0 \\ 21/25 \end{bmatrix} + \begin{bmatrix} 62/21 \\ 155/42 \\ 31/42 \\ 0 \end{bmatrix} = \begin{bmatrix} 457/105 \\ 2699/1050 \\ 31/42 \\ 21/25 \end{bmatrix}$

$\vec{y}_\perp = \vec{y} - \vec{y}_\parallel = \begin{bmatrix} 5 \\ 2 \\ 1 \\ -1 \end{bmatrix} - \begin{bmatrix} 457/105 \\ 2699/1050 \\ 31/42 \\ 21/25 \end{bmatrix} = \begin{bmatrix} 68/105 \\ -599/1050 \\ 11/42 \\ -46/25 \end{bmatrix}$