Instructions: Show all work. Use exact answers unless otherwise asked to round.

1. Suppose that the matrix below is the augmented matrix representing a system of equations.

$$\begin{bmatrix} 0 & 1 & 5 & -4 \\ 1 & 4 & ③ & -2 \\ 2 & 7 & 3 & -2 \end{bmatrix}$$

a. State the size of the matrix.

3×4

b. If we call the matrix A, what is the element A_{23}?

3

c. Write the system of equations represented by the matrix using the variables x_1, x_2,... as needed.

$$x_2 + 5x_3 = -4$$
$$x_1 + 4x_2 + 3x_3 = -2$$
$$2x_1 + 7x_2 + 3x_3 = -2$$

d. Solve the system by reducing the matrix, by hand, to *reduced* row echelon form.

$R_1 \leftrightarrow R_2$
$-2R_2 + R_3 \to R_3$
$\begin{bmatrix} 1 & 4 & 3 & -2 \\ 0 & 1 & 5 & -4 \\ 0 & -1 & -3 & 2 \end{bmatrix}$
$R_2 + R_3 \to R_3$
$\begin{bmatrix} 1 & 4 & 3 & -2 \\ 0 & 1 & 5 & -4 \\ 0 & 0 & 2 & -2 \end{bmatrix}$
$\frac{1}{2}R_3 \to R_3$
$\begin{bmatrix} 1 & 4 & 3 & -2 \\ 0 & 1 & 5 & -4 \\ 0 & 0 & 1 & -1 \end{bmatrix}$

$-5R_3 + R_2 \to R_2$
$-3R_3 + R_1 \to R_1$
$\begin{bmatrix} 1 & 4 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$
$-4R_2 + R_1 \to R_1$
$\begin{bmatrix} ① & 0 & 0 & -3 \\ 0 & ① & 0 & 1 \\ 0 & 0 & ① & -1 \end{bmatrix}$

$$x_1 = -3$$
$$x_2 = 1$$
$$x_3 = -1$$

$$\vec{x} = \begin{bmatrix} -3 \\ 1 \\ -1 \end{bmatrix}$$

e. Circle the pivots in your reduced matrix.
f. State whether the solution is consistent or inconsistent; dependent or independent. If independent, state the solution in vector (coordinate point) form. If dependent, use set notation to write the reduced set of equations the system must satisfy.

Consistent, independent