MTH 266, Quiz #8, Fall 2018 Name KEY

Instructions: Show all work. Use exact answers unless otherwise asked to round.

1. Let $A = \begin{bmatrix} 1 & -3 & -5 \\ 7 & -7 & 5 \end{bmatrix}$ and define $T: R^3 \to R^2$ by $T(\vec{x}) = A\vec{x}$.

   a. Find the image under T of $\vec{u} = \begin{bmatrix} 2 \\ -11 \end{bmatrix}$.

$$\begin{bmatrix} 1 & -3 & -5 \\ 7 & -7 & 5 \end{bmatrix}\begin{bmatrix} 2 \\ -11 \end{bmatrix} =$$ This operation is not defined

there is no image of $\vec{u}$ under T

   b. Find a vector whose image under T is $\vec{b} = \begin{bmatrix} 12 \\ -12 \end{bmatrix}$. Is it unique?

$$\begin{bmatrix} 1 & -3 & -5 \\ 7 & -7 & 5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 12 \\ -12 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & -3 & -5 & 12 \\ 7 & -7 & 5 & -12 \end{array}\right] \Rightarrow \text{rref} \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 25/7 & -60/7 \\ 0 & 1 & 20/7 & -48/7 \end{array}\right]$$

$x_1 = -25/7\, x_3 - 60/7$

$x_2 = -20/7\, x_3 - 48/7$

$x_3 = x_3$

if $x_3 = 1 \Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -85/7 \\ -68/7 \\ 1 \end{bmatrix}$ or if $x_3 = 0$ $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -60/7 \\ -48/7 \\ 0 \end{bmatrix}$

2. ~~Show that~~ Determine if the transformation $T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} 3x_2 - 4x_3 \\ 1 - x_1^2 \\ x_1 + x_2 - x_3 \end{bmatrix}$ is linear. ~~Determine if the transformation is linear~~. If it is not, explain why not. If it is, prove it. Is the transformation one-to-one, onto, both, or neither? Explain.

it is not linear (it fails all tests but in particular)

$$T(0) = \begin{bmatrix} 3(0) - 4(0) \\ 1 - (0)^2 \\ (0) + (0) - (0) \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \neq \vec{0}$$