

Instructions: Show all work. Use exact answers unless otherwise asked to round.

1. Find a basis for the space spanned by the vectors $\left\{ \begin{bmatrix} 9 \\ 3 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 4 \\ 2 \\ -3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 7 \\ 5 \\ 3 \\ 2 \end{bmatrix} \right\}$.

$$\begin{bmatrix} 9 & -1 & 0 & 7 \\ 3 & 4 & 1 & 5 \\ 2 & 2 & 1 & 3 \\ 1 & -3 & -1 & 2 \end{bmatrix} \Rightarrow \text{ref} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The set forms a basis for all of \mathbb{R}^4 .

2. Given the basis $\{2 - t, t + t^2, 3t^2 - t^3, 1 + 4t^3\}$ for P_3 , find the representation of $p(t) = 5t^2 - 3t + 17$ in this basis. Clearly label your change of basis matrix and correct notation for each vector used.

$$P_B [\vec{x}]_B = \vec{x} \quad \Rightarrow \quad P_B^{-1} \vec{x} = [\vec{x}]_B$$

$$P_B = \begin{bmatrix} 2 & 0 & 0 & 1 \\ -1 & 1 & 0 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & -1 & 4 \end{bmatrix}$$

$$P_B^{-1} \begin{bmatrix} 17 \\ -3 \\ 5 \\ 0 \end{bmatrix} = \begin{bmatrix} 196/23 \\ 127/23 \\ -4/23 \\ -1/23 \end{bmatrix}_B$$