MTH 267, Quiz #2, Fall 2018 Name KEY

Instructions: Show all work. Use exact answers unless otherwise asked to round.

1. Use Euler's method to find the value of $y(0.5)$ given the differential equation $\frac{dy}{dt} = -\frac{1}{3}xy + 1$ given the initial conditions $y(1) = 2$ in five steps. (Note: Δt is negative.)

$\Delta t = -0.1$ $x_0 = 1$ $y_0 = 2$ $m_0 = -\frac{1}{3}(1)(2) + 1 = \frac{1}{3}$ $y_1 = \frac{1}{3}(-0.1) + 2 = 1.96667$

$x_1 = 0.9$ $y_1 = 1.9667$ $m_1 = -\frac{1}{3}(0.9)(1.9667) + 1 = 0.41$ $y_2 = 0.41(-0.1) + 1.9667$
$= 1.92567$

$x_2 = 0.8$ $y_2 = 1.92567$ $m_2 = -\frac{1}{3}(0.8)(1.92567) + 1 = 0.486$ $y_3 = 0.468(-0.1) + 1.92567$
$= 1.87702$

$x_3 = 0.7$ $y_3 = 1.877$ $m_3 = -\frac{1}{3}(0.7)(1.877) + 1 = 0.562$ $y_4 = 0.562(-0.1) + 1.877$
$= 1.8208$

$x_4 = 0.6$ $y_4 = 1.8208$ $m_4 = -\frac{1}{3}(0.6)(1.8208) + 1 = 0.6358$ $y_5 = 0.6358(-0.1) + 1.8208$
$= 1.75723$

$\boxed{x_5 = 0.5 \quad y_5 = 1.75723}$

2. Verify that $y(x) = \frac{1}{\sqrt[3]{3\cos x + 8}}$ is a solution to the differential equation $\frac{dy}{dx} = y^4 \sin x$, $y\left(\frac{\pi}{2}\right) = \frac{1}{2}$.

$y = (3\cos x + 8)^{-1/3}$

$\frac{dy}{dx} = -\frac{1}{3}(3\cos x + 8)^{-4/3}(-3\sin x)$

$= \left(\frac{1}{\sqrt[3]{3\cos x + 8}}\right)^4 \cdot \sin x$

$\uparrow$

$y^4 \cdot \sin x$ ✓

$y\left(\frac{\pi}{2}\right) = \frac{1}{\sqrt[3]{3\cos(\pi/2) + 8}} = \frac{1}{\sqrt[3]{0+8}} = \frac{1}{2}$ ✓