

**Instructions:** Show all work. Use exact answers unless otherwise asked to round.

1. A system of differential equations is given by  $\begin{cases} \frac{dx}{dt} = -0.9x + 0.6x^2 - xy \\ \frac{dy}{dt} = 0.2y - 0.4y^2 - xy \end{cases}$ . Sketch the phase

plane using nullclines. Identify all equilibria. Use technology to determine which, if any, of the equilibria are stable. Include both your hand-drawn graph and the technology-based graph. Explain your reasoning for categorizing the equilibria.

$$0 = -0.9x + 0.6x^2 - xy$$

$$= x(-0.9 + 0.6x - y)$$

$$x=0 \quad 0 = -0.9 + 0.6x - y$$

$$y = 0.6x - 0.9$$

$$\frac{dy}{dt} = 0 = 0.2y - 0.4y^2 - xy$$

$$= y(0.2 - 0.4y - x)$$

$$y=0 \quad 0 = 0.2 - 0.4y - x$$

$$\frac{-x + 0.2}{0.4} = \frac{0.4y}{0.4}$$

$$-2.5x + 0.5 = y$$

$$0.6x - 0.9 = -2.5x + 0.5$$

$$3.1x = 1.4$$

$$x = \frac{1.4}{3.1} = \frac{14}{31} \approx 0.45$$

$$y = 0.6\left(\frac{14}{31}\right) - 0.9 = \frac{-39}{62} \approx -0.63$$

equilibria:  $(0,0)$  Saddle

Stable  $(0,0.5)$  ~~Saddle~~

$(1.5, 0)$  Saddle

$\left(\frac{14}{31}, \frac{-39}{62}\right)$  unstable



