MTH 267, Quiz #9, Fall 2018 Name KEY

Instructions: Show all work. Use exact answers unless otherwise asked to round.

1. Solve the system $\vec{x}' = \begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix}\vec{x}$ for the general solution.

$(3-\lambda)(2-\lambda)-2=0$

$\lambda^2-5\lambda+6-2=0$

$\lambda^2-5\lambda+4=0$

$(\lambda-4)(\lambda-1)=0$

$\lambda=4, \lambda=1$

$\lambda=4 \quad \begin{pmatrix} -1 & 2 \\ 1 & -2 \end{pmatrix} \quad x_1=2x_2 \quad \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$\lambda=1 \quad \begin{pmatrix} 2 & 2 \\ 1 & 1 \end{pmatrix} \quad x_1=-x_1 \quad \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$\vec{x} = c_1\begin{pmatrix} 2 \\ 1 \end{pmatrix}e^{4t} + c_2\begin{pmatrix} 1 \\ -1 \end{pmatrix}e^{t}$

2. A force of 50 N stretches a spring 0.8 meters. A dashpot device is attached that applies a force of 3 N for each unit of velocity. A mass of 10 kg is attached to the end of the spring and is initially released from equilibrium position with a downward velocity of 0.1 m/s. Write the second-order equation that models the system. Then convert it to a system of first-order equations.

$50 = k(.8)$

$k = 62.5$

$\gamma = 3$

$m = 10$

$10y'' + 3y' + 62.5y = 0$

$y(0) = 0$

$y'(0) = -0.1$

$y = x_1$

$x_1' = x_2$

$x_2' = -6.25x_1 - .3x_2$

$\vec{x}' = \begin{pmatrix} 0 & 1 \\ -6.25 & -.3 \end{pmatrix}\vec{x} \qquad \begin{pmatrix} 0 \\ -0.1 \end{pmatrix} = \vec{x}(0)$