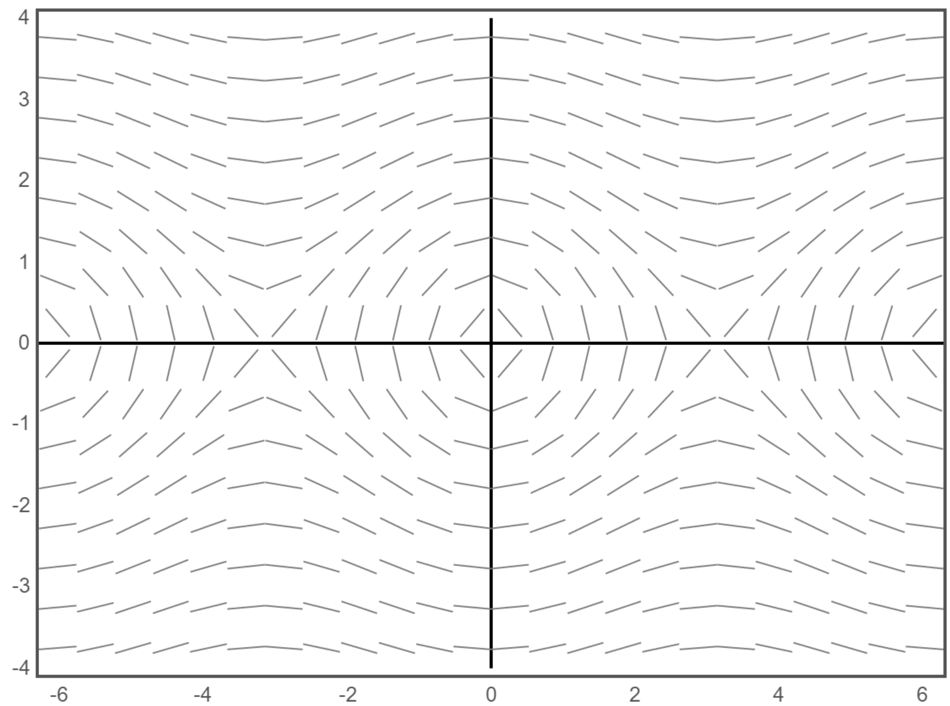
MTH 267, Practice Exam #1, Fall 2019   Name ____

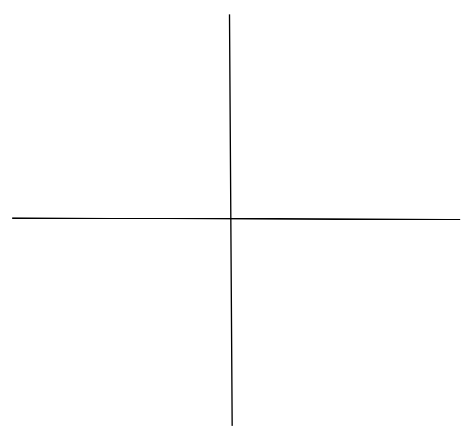
Instructions: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Verify that $y(x) = \ln(x + C)$ is a solution to the differential equation $e^y y' = 1, y(0) = 0$.

2. Shown below is the slope field for an undamped pendulum. Plot at least three sample trajectories (integral solutions) with different behaviors. Track the path forward and backward in time (so that the path begins and ends on the edges of the graph.



3. Use the Existence and Uniqueness Theorem to determine the regions where a solution to the ODE $y^2(xy' + y)\sqrt{1 + x^4} = x$ is guaranteed to exist. Sketch the region in the plane. Be sure to check all conditions and show your work.



4. Solve $x^2 y' = 1 - x^2 + y^2 - x^2 y^2$ by separation of variables. [Hint: Factor by grouping.]

5. Classify each differential equation as i) linear or nonlinear, ii) state its order.
   a. $x^2 y'' + 5xy' + 4y = 0$

b. $\frac{du}{dx} + \frac{d^2u}{dx^2} = xu$

c. $\frac{d^3y}{dt^3} + 2\cos t \frac{d^2y}{dt^2} + \left(\frac{dy}{dt}\right)^2 = e^t$

d. $u^{(5)} + u''' = \ln(u)$

6. Determine if the differential equation is homogeneous. Explain your reasoning.
   a. $x^2y'' + 2xy' - 6y = 0$

   b. $x^2y' + 2xy = 5x^4$

7. Determine the solution method for each of the following equations. Do not solve. Choose from: i) separable, ii) linear, iii) exact, iv) Bernoulli, v) homogeneous, vi) none of these.
   a. $x^2y' + 2xy = 5y^4$

   b. $y' + 2xy^2 = 0$

   c. $\frac{dy}{dx} = x^2 - y$

   d. $\frac{dy}{dx} = \frac{(x-1)y^5}{x^2(2y^3-y)}$

   e. $(1 + ye^{xy})dx + (2y + xe^{xy})dy = 0$

   f. $(x^2 - y^2)y' = 2xy$

8. Solve each of the problems in #7.

9. Use the method of integrating factors to find the particular solution for $xy' - 3y = 2x^4e^x$.

10. Solve the Bernoulli equation $x^2y' + 2xy = 5y^4$ as a linear equation.

11. Verify that the equation $(2x + y^2)dx + (2xy)dy = 0$ is exact. Then find the general solution.

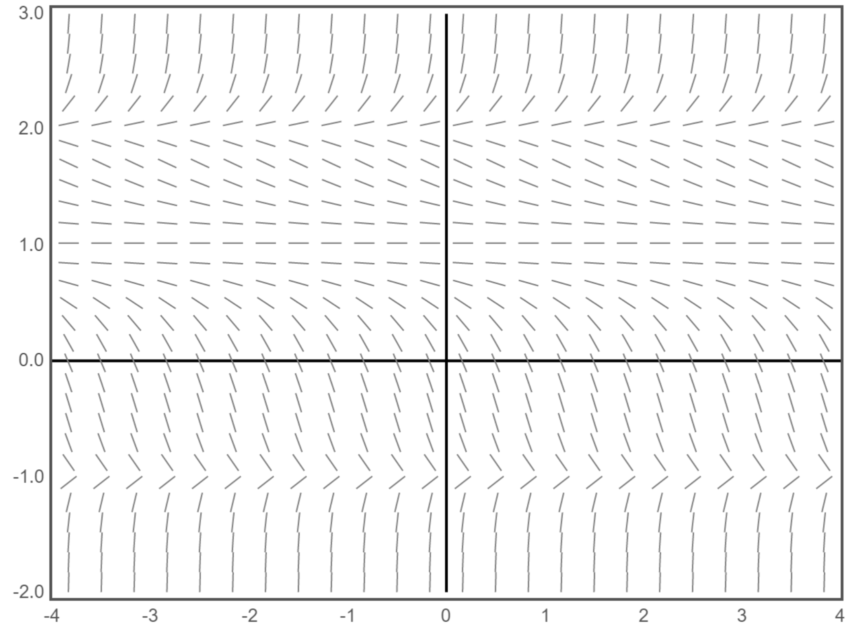
12. Solve the homogeneous equation $(x + 2y)y' = y$, and state its order.

13. Describe the conditions in a population model needed to label an equilibrium as a carrying capacity.

14. Use Euler's Method for 5 steps to approximate the solution of $xy' = y^2, y(1) = 1$ at the point $y(2)$.

15. Estimate the solution of the ODE $\frac{dy}{dx} = y\cos x, y(0) = 1$ using $\Delta t = 0.1$ using two complete steps of Runge-Kutta.

16. Consider the slope field shown below. If the equilibria are assumed to be integer values, write a differential equation that can produce the field. Characterize each equilibrium as stable, unstable or semistable.



17. Use an integrating factor to solve $e^x dx + (e^x \cot y + 2y \csc y)dy = 0$.

18. Find the value of k that will make the equation $(3x^2 - kxy + 2)dx + (6y^2 - x^2 + 3)dy = 0$ exact.