MTH 267, Practice Final Exam, Fall 2019 Name ____

Instructions: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Estimate the solution of the ODE $\frac{dy}{dx} = y - xy, y(0) = 2$ using $\Delta t = 0.1$ using two complete steps of Runge-Kutta.

2. Verify that the equation $(1 + ye^{xy})dx + (2y + xe^{xy})dy = 0$ is exact. Then find the general solution.

3. Use the method of integrating factors to find the particular solution for $xy' = 2y + x^3 \cos x$, $y(\pi) = 0$.

4. Rewrite the equation $y' + \frac{6}{x}y = 3y^{4/3}$ as a linear equation (Hint: it is Bernoulli).

5. Solve $y' + 2xy^2 = 0$ by separation of variables.

6. Classify each differential equation as i) linear or nonlinear, ii) state its order.
   a. $yy' = x(y^2 + 1)$
   b. $\frac{d^4y}{dx^4} = y \cos x$
   c. $2\sqrt{x}\frac{du}{dx} + \left(\frac{du}{dx}\right)^2 = 2xu$
   d. $y^{(5)} + y'' = e^y \tan x$

7. Solve the second-order ODEs for the general solution.
   a. $y'' - 2y' + 2y = 0$
   b. $2y'' - y' - 2y = 0$

8. The table below gives the solution to the second order constant coefficient homogeneous equation, and the forcing function $F(x)\ or\ F(t)$. Determine the Ansatz (particular solution y_p) for the method of undetermined coefficients in each case.

	y_1	y_2	$F(x)\ or\ F(t)$	Ansatz
a.	e^{-4x}	$e^{0.1x}$	$2 \sinh 3x$	
b.	$e^x \cos x$	$e^x \sin x$	$e^x \sin x$	
c.	e^x	$e^{-x/3}$	$e^x + 7x^3$	

9. Use the table of Laplace transforms to find Laplace transforms or inverse Laplace transforms as indicated. (4 points each)
   a. $\mathcal{L}\{(1+2t)^2\}$
   b. $\mathcal{L}\{e^{-2t}\sin 3t\}$
   c. $\mathcal{L}\left\{\frac{1}{2}\int_0^t (t-\tau)^3 \sin 2\tau \, d\tau\right\}$
   d. $\mathcal{L}^{-1}\left\{\frac{1}{2}-\frac{2}{s^5}\right\}$
   e. $\mathcal{L}^{-1}\left\{\frac{9-17s}{s^2+81}\right\}$
   f. $\mathcal{L}^{-1}\left\{\frac{1}{(s-3)(s^2+4)(s^2-1)}\right\}$
   g. $\mathcal{L}^{-1}\left\{\frac{e^{-\pi}}{s^2+1}\right\}$
10. Use Laplace transforms to solve the IVP $y''+4y'-12y=e^{-2t}, y(0)=0,\ y'(0)=1$.
11. We want to approximate the solution to $y'=x+\sqrt[3]{y}$ at the point $x=3$ in 10 steps. Given that $y(0)=1$, compute the first 3 steps of the approximation with Euler's method.
12. A 1000L tank initially contains only pure water. A hose begins adding to the tank at a rate of 5L/min with a concentration of iodine salt of 40g/L. The well-mixed solution flows out of the tank at a rate of 6L/min. Find an equation that models the amount of iodine in the tank after time t. Find the maximum amount of iodine in the tank (if one exists).
13. Determine if the set of functions forms a fundamental set.
   a. $e^t \sin t, e^t \cos t$
   b. $\cosh t, \sinh t$
14. Use reduction of order to find the second solution to the equation $(x-1)y''-xy'+y=0$, $y_1=e^x$.
15. Find the particular solution $y''+2y'+5y=3\sin 2t, y(0)=1, y'(0)=3$ using:
   a. The method of undetermined coefficients
   b. Variation of parameters
16. A spring with a 4-kg mass has natural length 1 m and is maintained stretched to a length of 1.3 m by a force of 24.3 N. If the spring is compressed to a length of 0.8 m and then released with zero velocity, set up the second order linear IVP needed to solve the system, then solve it. You may round solutions to 4 decimal places.
17. Use the definition of the Laplace transform $\mathcal{L}\{f(t)\}=\int_0^\infty e^{-st}f(t)dt$ to find $\mathcal{L}\{1+\cosh 5t\}$.

18. Use the table of Laplace transforms to find Laplace transforms or inverse Laplace transforms as indicated.
    a. $\mathscr{L}\{(1+t)^2\}$

    b. $\mathscr{L}\{te^t\}$

    c. $\mathscr{L}\{e^{-2t}\sin 3\pi t\}$

    d. $\mathscr{L}^{-1}\left\{\frac{1}{2}-\frac{2}{s^5}\right\}$

    e. $\mathscr{L}^{-1}\left\{\frac{9-17s}{s^2+81}\right\}$

    f. $\mathscr{L}^{-1}\left\{\frac{1}{s(s^2+4)}\right\}$

    g. $\mathscr{L}^{-1}\left\{\frac{1}{s^2(s^2-1)}\right\}$

19. Use Laplace transforms to solve the IVP $y''+4y'+8y=e^{-t}, y(0)=0,\ y'(0)=1$.

20. Use the definition of the Laplace transform $\mathscr{L}\{f(t)\}=\int_0^\infty e^{-st}f(t)dt$ to find a formula for $\mathscr{L}\{f(t)\}$.

    a. $f(t)=t^2$
    b. $f(t)=\cosh 2t$
    c. $f(t)=\begin{cases}1, & 0\le t\le 2\\ 2t-1, & 2>t\end{cases}$

21. A metal pan is removed from an oven at a temperature of 425-degrees. After 2 minutes, the pan temperature has fallen to 350-degrees.
    a. If the room temperature is 77-degrees, write a differential equation that models the situation, and then solve for the equation at time *t*.
    b. How long will it take for the temperature to fall to 120-degrees?