MTH267 Practice Final Exam Key

1. $\frac{dy}{dx} = y - xy \quad y(0) = 2 \quad \Delta t = 0.1$

$n=0 \quad x_0 = 0, \; y_0 = 2$

$k_{01} = 0.1(2 - 0(2)) = 0.2$

$k_{02} = 0.1(2.1 - 0.05(2.1)) = 0.1995$

$k_{03} = 0.1(2.09975 - 0.05(2.09975)) = 0.18947625$

$k_{04} = 0.1(2.19947625 - 0.1(2.19947625)) = 0.197952860$

$y_{n+1} = 2 + 0.199317504 = 2.1993$

$n=1 \quad x_1 = 0.1 \quad y_1 = 2.1993$

$k_{11} = 0.1(2.1993 - 0.1(2.1993)) = 0.197938\ldots$

$k_{12} = 0.1(2.098969 - 0.15(2.098969)) = 0.1784123\ldots$

$k_{13} = 0.1(2.08921 - 0.15(2.08921)) = 0.1775825\ldots$

$k_{14} = 0.1(2.1775 - 0.2(2.1775)) = 0.1742066\ldots$

$y_2 = 2.1993 + 0.180689 = \boxed{2.379989}$

2. $\frac{\partial M}{\partial y} = e^{xy} + xye^{xy} \qquad \frac{\partial N}{\partial x} = e^{xy} + xye^{xy}$

$\int 1 + ye^{xy}\,dx = x + e^{xy} + f(y)$

$\int 2y + xe^{xy}\,dy = y^2 + e^{xy} + g(x)$

$\varphi(x,y) = x + y^2 + e^{xy} + K$

3. $xy' - 2y = x^3 \cos x$

$y' - \frac{2}{x}y = x^2 \cos x$

$\mu = e^{\int -\frac{2}{x}dx} = e^{-2\ln x} = x^{-2}$

$x^{-2}y' - 2x^{-3}y = \cos x$

$\int (x^{-2}y)' = \int \cos x\,dx$

$x^{-2}y = \sin x + C$

$y = x^2 \sin x + Cx^2$

$y(\pi) = 0$

$0 = \cancel{\pi^2 \sin\pi}^{\,0} + C\pi^2$

$C = 0$

$y(x) = x^2 \sin x$

4. $y' + \frac{6}{x}y = 3y^{4/3}$

(This equation is Bernoulli)

$(1-n)y^{-n} = (1-\frac{4}{3})y^{-4/3} = -\frac{1}{3}y^{-4/3}$

$-\frac{1}{3}y^{-4/3}y' - \frac{2}{x}\left(-\frac{1}{3}\right)y^{-1/3} = -1$

let $z = y^{-1/3}$

$z' = \frac{-1}{3}y^{-4/3}y'$

$z' + \frac{2}{3x}z = -1$

this equation is linear

5. $y' + 2xy^2 = 0 \Rightarrow y' = -2xy^2 \Rightarrow \int \frac{dy}{y^2} = \int -2x\,dx$

$\Rightarrow -\frac{1}{y} = -x^2 + C \Rightarrow y = \frac{1}{x^2+C}$

6. a. nonlinear, 1st order   b. linear, 4th order

c. nonlinear, 1st order   d. nonlinear, 5th order

7. a. $r^2 - 2r + 2 = 0 \qquad r = \frac{2 \pm \sqrt{4-8}}{2} = 1 \pm i$

$y(t) = c_1 e^t \cos t + c_2 e^t \sin t$

b. $2r^2 - r - 2 = 0 \qquad r = \frac{1 \pm \sqrt{1+16}}{2(2)} = \frac{1}{4} \pm \frac{\sqrt{17}}{4}$

$y(t) = c_1 e^{[(1+\sqrt{17})/4]t} + c_2 e^{[(1+\sqrt{17})/4]t}$

8. a. $A \sinh 3x + B \cosh 3x = Y(x)$ Ansatz $= Y_P$

b. $Axe^x \cos x + Bxe^x \sin x = Y(x)$

c. $Axe^x + Bx^3 + Cx^2 + Dx + E = Y(x)$

9. a. $\mathcal{L}\{1 + 4t + 4t^2\} = \frac{1}{s} + \frac{4}{s^2} + \frac{8}{s^3}$

b. $\mathcal{L}\{e^{-2t} \sin 3t\} = \frac{3}{(s+2)^2 + 9}$

c. $\mathcal{L}\{\frac{1}{2}\int_0^t (t-\tau)^3 \sin 2\tau \, d\tau\} = \frac{1}{2}\mathcal{L}\{t^3\} \cdot \mathcal{L}\{\sin 2t\} =$

$\frac{1}{2} \cdot \frac{6}{s^4} \cdot \frac{2}{s^2+4} = \frac{6}{s^4(s^2+4)}$

d. $\mathcal{L}^{-1}\{\frac{1}{2} - \frac{2}{s^5}\} = \frac{1}{2}\delta(t) - \frac{1}{12}t^4$

e. $\mathcal{L}^{-1}\{\frac{9}{s^2+81} - \frac{17s}{s^2+81}\} = \sin 9t - 17\cos 9t$

f. $\mathcal{L}^{-1}\{\frac{1}{(s-3)(s^2+4)(s^2-1)}\} \Rightarrow \frac{A}{s-3} + \frac{Bs+C}{s^2+4} + \frac{Ds+E}{s^2-1}$

$A(s^2+4)(s^2-1) + (Bs+C)(s-3)(s^2-1) + (Ds+E)(s-3)(s^2+4) =$

$As^4 + 3As^2 - 4A + Bs^4 - 3Bs^3 - Bs^2 + 3Bs + Cs^3 - 3Cs^2 - Cs + 3C + Ds^4 - 3Ds^3$

$- 4Ds^2 - 12Ds + Es^3 - 3Es^2 + 4Es - 12E =$

9f cont'd

$A + B + D = 0 \quad (s^4)$

$3A - B - 3C + 4D - 3E = 0 \quad (s^2)$

$-4A + 3C - 12 = 1 \quad (1)$

$-3B - 3D + E = 0 \quad (s^3)$

$3B - C - 12D + 4E = 0 \quad (s)$

$A = \frac{5}{196}$

$B = \frac{1}{245}$

$C = \frac{3}{49}$

$D = \frac{-29}{980}$

$E = \frac{-15}{196}$

$$\mathcal{L}^{-1}\left\{ \frac{5/196}{s-3} + \frac{1/245\, s}{s^2+4} + \frac{3/49}{s^2+4} - \frac{29/980\, s}{s^2-1} - \frac{15/196}{s^2-1} \right\}$$

$$= \frac{5}{196} e^{3t} + \frac{1}{245} \cos 2t + \frac{3}{98} \sin 2t - \frac{29}{980} \cosh t - \frac{15}{196} \sinh t$$

g. $\mathcal{L}^{-1}\left\{ \frac{e^{-\pi}}{s^2+1} \right\} = e^{-\pi t} \sin t$

10. $y'' + 4y' - 12y = e^{-2t} \qquad y(0) = 0, \; y'(0) = 1$

$$\left(s^2 Y(s) - s(0) - 1\right) + 4\left(sY(s) - 0\right) - 12\, Y(s) = \frac{1}{s+2}$$

$$Y(s)\left(s^2 + 4s - 12\right) = \frac{1 + s + 2}{s+2} = \frac{s+3}{s+2}$$

$$Y(s) = \frac{s+3}{(s+2)(s+6)(s-2)} = \frac{A}{s+2} + \frac{B}{s+6} + \frac{C}{s-2}$$

$$As^2 + 4As - 12A + Bs^2 - 4B + Cs^2 + 8Cs + 12C = s + 3$$

$A + B + C = 0$

$4A + 8C = 1$

$-12A - 4B + 12C = 3$

$A = -\frac{1}{16}$

$B = \frac{-3}{32}$

$C = \frac{5}{32}$

$$Y(s) = \frac{-\frac{1}{16}}{s+2} - \frac{3/32}{s+6} + \frac{5/32}{s-2}$$

$$y(t) = -\frac{1}{16} e^{-2t} - \frac{3}{32} e^{-6t} + \frac{5}{32} e^{2t}$$

1. $y' = x + \sqrt[3]{y} \qquad y(0) = 1 \qquad \Delta x = \frac{3-1}{10} = \frac{2}{10} = \frac{1}{5} = 0.2$

$n=0 \quad x_0 = 0,\; y_0 = 1 \quad m_0 = (0 + \sqrt[3]{1})(0.2) = 0.2 \qquad y_1 = 1.2$

$n=1 \quad x_1 = 0.2,\; y_1 = 1.2 \quad m_0 = (0.2 + \sqrt[3]{1.2})(0.2) = 0.25253 \qquad y_2 = 1.2 + 0.2525(0.2) = 1.2505$

$n=2 \quad x_2 = 0.4 \quad y_2 = 1.2505 \quad m_2 = (0.4 + \sqrt[3]{1.2505})(0.2) = 0.29547\ldots \qquad y_3 = 1.30959\ldots$

12. $A(0)=0$ 4

$$\frac{dA}{dt} = \frac{5K}{min} \cdot \frac{40g}{K} - \frac{A}{100-t} \cdot \frac{6L}{min}$$

$$\mu = \int \frac{6}{1000-t}\,dt = e^{-6\ln(1000-t)} = (1000-t)^{-6}$$

$$\frac{dA}{dt} = 200 - \frac{6A}{1000-t} \Rightarrow A' = \frac{6}{1000-t} A = 200$$

$$(1000-t)^{-6}A' + 6(1000-t)^{-7}A = 200(1000-t)^{-6}$$

$$\int \left((1000-t)^{-6}A\right)' = \int 200(1000-t)^{-6}\,dt$$

$$(1000-t)^{-6}A = \frac{200}{5}(1000-t)^{-5} + C$$

$$A = 40(1000-t) + C(1000-t)^{6}$$

last for 1000 seconds until tank is empty

$$A(0)=0 = 40(1000-0) + C(1000-0)^{6}$$

$$\Rightarrow 0 = 40{,}000 + C \cdot 10^{18} \Rightarrow C = \frac{-40{,}000}{10^{18}} \Rightarrow C = -4\times 10^{-14}$$

$$A(t) = 40{,}000 - 40t - 4\times 10^{-14}(1000-t)^{6}$$

When the tank drains, the amount will be 0 g.

13. a. $\begin{vmatrix} e^t \sin t & e^t \cos t \\ e^t \sin t + e^t \cos t & e^t \cos t - e^t \sin t \end{vmatrix} =$

$$e^{2t}\sin t\,\cancel{\cos t} - e^{2t}\sin^2 t - e^{2t}\,\cancel{\sin t}\cos t - e^{2t}\cos^2 t = -e^{2t}(\sin^2 t + \cos^2 t)$$
$$= e^{2t}$$

b. $\begin{vmatrix} \cosh t & \sinh t \\ \sinh t & \cosh t \end{vmatrix} = \cosh^2 t - \sinh^2 t = 1$

14. $(x-1)y'' - xy' + y = 0 \qquad y_1 = e^x \qquad y_2 = e^x v$

$$y_2' = e^x v + e^x v'$$
$$y_2'' = e^x v + 2e^x v' + e^x v''$$

$$(x-1)(e^x v + 2e^x v' + e^x v'') -$$
$$x(e^x v + e^x v') + e^x v = 0$$

$$e^x[\cancel{xv} + 2xv' + xv'' - \cancel{v} - 2v' - v'' - \cancel{xv} - xv' + \cancel{v}] = 0$$

$$v''(x-1) + v'(x-2) = 0 \qquad u = v' \qquad \frac{du}{dx} = v''$$

$$x-1\,\overline{)\,-x+2} = -1,\quad -x+2 - (-x+1) = 1$$

$$u'(x-1) = -(x-2)u$$

$$\int \frac{du}{u} = \int -\frac{(x-2)}{x-1}\,dx = \int -1 + \frac{1}{x-1}\,dx \Rightarrow$$

$$\ln u = -x + \ln(x-1) = \ln e^{-x} + \ln(x-1) = \ln[e^{-x}(x-1)]$$

$$\Rightarrow u = (x-1)e^{-x} \qquad v = \int (x-1)e^{-x}\,dx = -xe^{-x}$$

$$y_2 = -xe^{-x} \cdot e^{x} = -x$$

$$y(x) = c_1 e^{x} + c_2 x$$

15. $y'' + 2y' + 5y = 0$

$r^2 + 2r + 5 = 0$

$$r = \frac{-2 \pm \sqrt{4-20}}{2} = -1 \pm 2i$$

$$y_1 = e^{-t}\cos 2t \qquad y_2 = e^{-t}\sin 2t$$

a. $Y(t) = A\sin 2t + B\cos 2t \qquad Y'(t) = 2A\cos 2t - 2B\sin 2t$

$Y''(t) = -4A\sin 2t - 4B\cos 2t$

$y(0) = 1,\ y'(0) = 3$

$$-4A\sin 2t - 4B\cos 2t + 4A\cos 2t - 4B\sin 2t - 5A\sin 2t + 5B\cos 2t = 3\sin 2t$$

$\sin 2t \quad (-4A - 4B + 5A) = 3 \qquad \cancel{A} - 4B = 3 \qquad A = 3/17$

$\cos 2t \quad (-4B + 4A + 5B) = 0 \qquad 4A + B = 0 \qquad B = -12/17$

$$y_p(t) = c_1 e^{-t}\cos 2t + c_2 e^{-t}\sin 2t + \frac{3}{17}\sin 2t - \frac{12}{17}\cos 2t$$

$$y(0) = 1 = c_1(1)(1) + \cancel{c_2(1)(0)}^{0} + \cancel{\frac{3}{17}(0)}^{0} - \frac{12}{17}(1) \Rightarrow c_1 = \frac{29}{17}$$

$$y'(t) = -\frac{29}{17}e^{-t}\cos 2t - \frac{58}{17}e^{-t}\sin 2t - c_2 e^{-t}\sin 2t + 2c_2 e^{-t}\cos 2t + \frac{6}{17}\cos 2t + \frac{24}{17}\sin 2t$$

$$y'(0) = 3 = -\frac{29}{17}(1)(1) - \cancel{\frac{58}{17}(1)(0)}^{0} - \cancel{c_2(1)(0)}^{0} + 2c_2(1)(1) + \frac{16}{17}(1) + \cancel{\frac{24}{17}(0)}^{0}$$

$$\Rightarrow \frac{64}{17} = 2C_2 \Rightarrow C_2 = \frac{32}{17}$$

$$y(t) = \frac{29}{17}e^{-t}\cos 2t + \frac{32}{17}e^{-t}\sin 2t + \frac{3}{17}\sin 2t - \frac{12}{17}\cos 2t$$

b. $$Y(t) = -e^{-t}\cos 2t \int \frac{3\sin 2t \cdot e^{-t}\sin 2t}{2e^{-2t}}\,dt + e^{-t}\sin 2t \int \frac{3\sin 2t\, e^{-t}\cos 2t}{2e^{-2t}}\,dt$$

$$= -\frac{3}{2}e^{-t}\cos 2t \int \sin^2 2t\, e^{t}\,dt + \frac{3}{2}e^{-t}\sin 2t \int \sin 2t\cos 2t\, e^{t}\,dt$$

$$= -\frac{3}{2}e^{-t}\cos 2t\left[-\frac{4}{17}e^{t}\sin 2t\cos 2t + \frac{1}{17}e^{t}\sin^2 2t + \frac{8}{17}e^{t}\right] +$$

$$\frac{3}{2}e^{-t}\sin 2t\left[\frac{-4}{34}e^{t}\cos 4t + \frac{1}{34}\sin 4t\, e^{t}\right] =$$

$= \frac{6}{17}\cos^2 2t \sin 2t - \frac{3}{2}\cos 2t \sin^2 2t - \frac{12}{17}\cos 2t - \frac{3}{17}\sin 2t \cos 4t$

$+ \frac{3}{68}\sin 4t \sin 2t$

$= \frac{6}{17}\cos^2 2t \sin 2t - \frac{3}{2}\cos 2t \sin^2 2t - \frac{12}{17}\cos 2t - \frac{3}{17}\sin 2t(\cos^2 2t - \sin^2 2$

$+ \frac{3}{68}\cdot 2\sin 2t \cos 2t \sin 2t$

$= \frac{6}{17}\cos^2 2t \sin 2t - \frac{3}{17}\sin 2t \cos^2 2t - \frac{3}{2}\cos 2t \sin^2 2t + \frac{3}{17}\sin 2t \sin^2 2t$

$+ \frac{3}{34}\sin^2 2t \cos 2t$

$= \frac{3}{17}\cos^2 2t \sin 2t + \frac{3}{17}\sin^2 2t \sin 2t - \frac{12}{17}\cos 2t - \frac{24}{17}\cos 2t \sin^2 2t$

$= \frac{3}{17}\sin 2t - \frac{12}{17}\cos 2t$ same solution as above, finish as of parta

16. $24.3 = k(.3) \Rightarrow k = 81 \quad m = 4$ $\quad y(0) = -0.2$

$y'(0) = 0$

$4y'' + 81y = 0$

$4r^2 + 81 = 0$

$r = \pm \frac{9}{2}i$

$y(t) = c_1 \sin(\frac{9}{2}t) + c_2(\cos \frac{9}{2}t)$

$y'(t) = \frac{9}{2}c_1 \sin(\frac{9}{2}t) - \frac{9}{2}c_2 \sin(\frac{9}{2}t)$

$y(0) = -0.2 = \cancel{c_1(0)}^{0} + c_2(1)$

$\Rightarrow c_2 = -0.2$

$y'(0) = \frac{9}{2}c_1(1) - \frac{9}{2}\cancel{(-0.2)(0)}^{0} = 0$

$c_1 = 0$

$y(t) = -0.2\cos(9/2\, t)$

17. $\int_0^\infty e^{-st}(1 + \cos 5t)\,dt = \int_0^\infty e^{-st} + e^{-st}\left(\frac{e^{5t} + e^{-5t}}{2}\right)dt =$

$\int_0^\infty e^{-st} + \frac{1}{2}e^{-t(s-5)} + \frac{1}{2}e^{-t(s+5)}\,dt =$

$-\frac{1}{s}e^{-st} - \frac{1}{2(s-5)}e^{-t(s-5)} - \frac{1}{2(s+5)}e^{-t(s+5)}\Big|_0^\infty = 0 + \frac{1}{s} + \frac{1}{2(s-5)} + \frac{1}{2(s+5)}$

$= \frac{1}{s} + \frac{s+5+s-5}{2(s+5)(s-5)} = \frac{1}{s} + \frac{\cancel{2}s}{\cancel{2}(s^2-25)} = \frac{1}{s} + \frac{s}{s^2-25}$

18a. $\mathcal{L}\{(1+t^2)\} = \frac{1}{s} + \frac{2!}{s^{2+1}} = \frac{1}{s} + \frac{2}{s^3}$

b. $\mathcal{L}\{te^t\} = \frac{1}{(s-1)^{1+1}} = \frac{1}{(s-1)^2}$

18 cont'd

c. $\mathcal{L}\{e^{-2t}\sin 3\pi t\} = \frac{s+2}{(s+2)^2+9\pi^2}$

d. $\mathcal{L}^{-1}\{\frac{1}{2} - \frac{2}{s^5}\} = \frac{1}{2}\delta(t) = \frac{1}{12}t^4$

e. $\mathcal{L}^{-1}\{\frac{9-17s}{s^2+81}\} = \mathcal{L}^{-1}\{\frac{9}{s^2+81}\} - 17\mathcal{L}^{-1}\{\frac{s}{s^2+81}\} = \sin 9t - 17\cos 9t$

f. $\mathcal{L}^{-1}\{\frac{1}{s(s^2+4)}\} = \mathcal{L}^{-1}\{\frac{1}{s}\cdot\frac{1}{s^2+4}\} = \int_0^t 1\cdot\frac{1}{2}\cdot\sin(2(t-\tau))\,d\tau$

g. $\mathcal{L}^{-1}\{\frac{1}{s^2(s^2-1)}\} = \mathcal{L}^{-1}\{\frac{1}{s^2}\cdot\frac{1}{s^2-1}\} = \int_0^t t\cdot\sinh(t-\tau)\,d\tau$

19. $y'' + 4y' + 8y = e^{-t} \qquad y(0)=0,\ y'(0)=1$

$(s^2Y(s) - s(0) - 1) + 4(sY(s) - 0) + 8Y(s) = \frac{1}{s+1}$

$s^2Y(s) - 1 + 4sY(s) + 8Y(s) = \frac{1}{s+1}$

$Y(s)(s^2+4s+8) = \frac{1}{s+1} + 1 = \frac{1+s+1}{s+1} = \frac{s+2}{s+1}$

$Y(s) = \frac{s+2}{(s+1)(s^2+4s+8)} = \frac{A}{s+1} + \frac{Bs+C}{s^2+4s+8}$

$(s+2)^2+4$

$As^2 + 4As + 8A + Bs^2 + Cs + Bs + C = s+2$

$A+B=0 \qquad A = 1/5$

$4A+B+C=1 \qquad B=-1/5$

$8A+C=2 \qquad C = 2/5$

$Y(s) = \frac{1/5}{s+1} + \frac{-1/5\, s}{(s+2)^2+4} + \frac{2/5}{(s+2)^2+4} \qquad -\frac{1}{5}[(s+2)-2] = -\frac{1}{5}(s+2) + \frac{2}{5}$

$= \frac{1/5}{s+1} + \frac{-1/5\,(s+2)}{(s+2)^2+4} + \frac{4/5}{(s+2)^2+4}$

$y(t) = \frac{1}{5}e^{-t} - \frac{1}{5}e^{-2t}\cos 2t + \frac{4}{5}e^{-2t}\sin 2t$

20 a. $\int_0^\infty e^{-st}t^2\,dt$

±	u	dv
+	t^2	e^{-st}
−	$2t$	$-\frac{1}{s}e^{-st}$
+	2	$\frac{1}{s^2}e^{-st}$
−	0	$-\frac{1}{s^3}e^{-st}$

$= -\frac{t^2}{s}e^{-st} - \frac{2t}{s^2}e^{-st} - \frac{2}{s^3}e^{-st}\Big|_0^\infty = 0+0+0+0+0+\frac{2}{s^3} \Rightarrow \frac{2}{s^3}$

20b.

$$\int_0^\infty e^{-st}\cosh 2t\,dt = \int_0^\infty e^{-st}\left(\frac{e^{2t}+e^{-2t}}{2}\right)dt = \frac{1}{2}\int_0^\infty e^{-t(s-2)} + e^{-t(s+2)}\,dt$$

$$= \frac{1}{2}\left[-\frac{1}{s-2}e^{-t(s-2)} - \frac{1}{s+2}e^{-t(s+2)}\right]_0^\infty = +\frac{1}{2(s-2)} + \frac{1}{2(s+2)} =$$

$$\frac{+s+2+s-2}{2(s-2)(s+2)} = \frac{\cancel{2}s}{\cancel{2}(s-2)(s+2)} = \frac{s}{s^2-4}$$

c. $$\int_0^2 1\,e^{-st}\,dt + \int_2^\infty (2t-1)e^{-st}\,dt = \int_0^2 e^{-st}\,dt + 2\int_2^\infty te^{-st}\,dt - \int_2^\infty e^{-st}\,dt$$

$$= -\frac{1}{s}e^{-st}\Big|_0^2 + 2\left[-\frac{t}{s}e^{-st} - \frac{1}{s^2}e^{-st}\right|_2^\infty + \frac{1}{s}e^{-st}\Big|_2^\infty =$$

$\pm$	u	dv
$+$	t	e^{-st}
$-$	1	$-\frac{1}{s}e^{-st}$
	0	$\frac{1}{s^2}e^{-st}$

$$-\frac{1}{s}e^{-2s} + \frac{1}{s} + 2\left[0 + \frac{2}{s}e^{-2s} - 0 + \frac{1}{s^2}e^{-2s}\right] + 0 - \frac{1}{s}e^{-2s}$$

$$= -\frac{1}{s}e^{-2s} + \frac{1}{s} + \frac{4}{s}e^{-2s} + \frac{1}{s^2}e^{-2s} - \frac{1}{s}e^{-2s}$$

$$= e^{-2s}\left(-\frac{1}{s} + \frac{4}{s} + \frac{1}{s^2} - \frac{1}{s}\right) + \frac{1}{s} = e^{-2s}\left(\frac{2}{s} + \frac{1}{s^2}\right) + \frac{1}{s}$$

21. $T(0) = 425$ $\qquad \frac{dT}{dt} = k(T-77)$

$T(2) = 350$

$T_R = 77$ $\qquad \frac{dT}{T-77} = k\,dT \Rightarrow \ln|T-77| = kt + C$

$\Rightarrow T - 77 = T_i e^{kt}$

$T(t) = 77 + T_i e^{kt}$

$T(0) = 425 = 77 + T_i \Rightarrow T(t) = 77 + 348e^{kt}$

$T(2) = 77 + 348e^{2k} = 350$

$273 = 348e^{2k} \Rightarrow k = -0.121365$

$T(\tau) = 120 = 77 + 348e^{-0.12136t} \Rightarrow t = 17.22899$ minutes