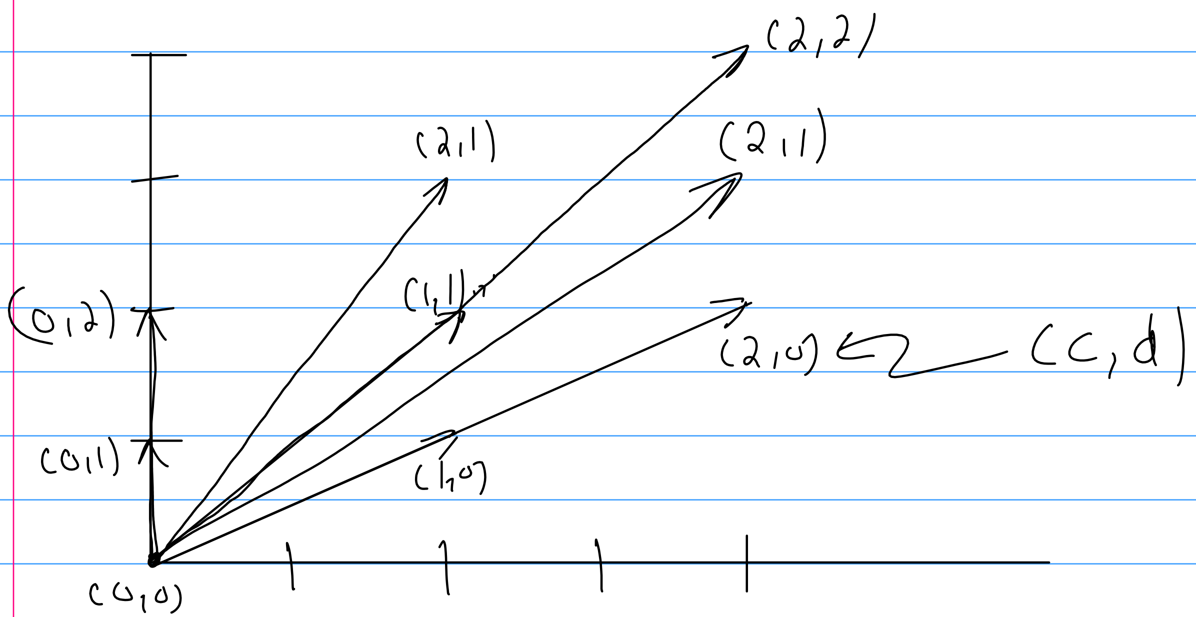
HW 2 P1

$c\begin{bmatrix}2\\1\end{bmatrix} + d\begin{bmatrix}0\\1\end{bmatrix}$ for $c = 0, 1, 2$
$d = 0, 1, 2$



HW 2   P2

$\underline{v} + \underline{w} = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$ $\underline{v} - \underline{w} = \begin{bmatrix} 2 \\ 5 \\ 9 \end{bmatrix}$

$\Rightarrow$ $v_1 + w_1 = 4$ $v_1 - w_1 = 2$ (1)

$v_2 + w_2 = 5$ $v_2 - w_2 = 5$ (2)

$v_3 + w_3 = 6$ $v_3 - w_3 = 8$ (3)

6 unknown numbers

(1) $v_1 = 4 - w_1 \Rightarrow 4 - w_1 - w_1 = 2 \Rightarrow w_1 = 1$

$v_1 = 3$

(2) $v_2 = 5 - w_2 \Rightarrow 5 - w_2 - w_2 = 5 \Rightarrow w_2 = 0$

$v_2 = 5$

(3) $v_3 = 6 - w_3 \Rightarrow 6 - w_3 - w_3 = 8 \Rightarrow w_3 = -1$

$v_3 = 7$

$\underline{v} = \begin{bmatrix} 3 \\ 5 \\ 7 \end{bmatrix}$ $\underline{w} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

Check (not needed for 2 pts)

$v + w = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$ $v - w = \begin{bmatrix} 2 \\ 5 \\ 9 \end{bmatrix}$ ✓✓

HW2 P3

$\underline{v} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ $\underline{w} = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$

$\underline{u}_1$: $\|\underline{v}\| = (3^2+1^2)^{1/2} = \sqrt{10}$

$$\underline{u}_1 = \frac{\underline{v}}{\|\underline{v}\|} = \frac{1}{\sqrt{10}}\begin{bmatrix} 3 \\ 1 \end{bmatrix} \approx \begin{bmatrix} 0.9487 \\ 0.3162 \end{bmatrix} = \underline{u}_1$$

Vector perp to $\underline{u}_1$: $\underline{U}_1 = \pm\begin{bmatrix} -0.3162 \\ 0.9487 \end{bmatrix}$

$\underline{u}_2$: $\|\underline{w}\| = (2^2+1^2+2^2)^{1/2} = 3$

$$\underline{u}_2 = \frac{\underline{w}}{\|\underline{w}\|} = \frac{1}{3}\begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2/3 \\ 1/3 \\ 2/3 \end{bmatrix} = \underline{u}_2$$

Vectors perp to $\underline{u}_2$:

$$\frac{a}{\sqrt{5}}\begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix} + \frac{b}{\sqrt{2}}\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + \frac{c}{\sqrt{5}}\begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix}$$ for any a, b, c

HW 2 P4

$\|\underline{v}\| = 5 \quad \|\underline{w}\| = 3$

$\|\underline{v} - \underline{w}\|$:

$\|\underline{v} - \underline{w}\|^2 = (\underline{v} - \underline{w}) \cdot (\underline{v} - \underline{w})$

$= \|\underline{v}\|^2 + \|\underline{w}\|^2 - 2\, \underline{v} \cdot \underline{w}$

$= \|\underline{v}\|^2 + \|\underline{w}\|^2 - 2\|\underline{v}\| \|\underline{w}\| \cos\theta$

As $-1 \le \cos\theta \le 1$

$\|\underline{v}\|^2 + \|\underline{w}\|^2 - 2\|\underline{v}\|\|\underline{w}\| \le \|\underline{v} - \underline{w}\|^2 \le \|\underline{v}\| + \|\underline{w}\| + 2\|\underline{v}\|\|\underline{w}\|$

$5^2 + 3^2 - 2(3)(5) \le \|\underline{v} - \underline{w}\|^2 \le 5^2 + 3^2 + 2(3)(5)$

$4 \le \|\underline{v} - \underline{w}\|^2 \le 64$

$\Rightarrow \quad 2 \le \|\underline{v} - \underline{w}\| \le 8$

min $\|\underline{v} - \underline{w}\| = 2$

max $\|\underline{v} - \underline{w}\| = 8$

min/max $\underline{u} \cdot \underline{v}$? $\quad \underline{u} \cdot \underline{v} = \|\underline{u}\| \, \|\underline{v}\| \cos\theta$

$\Rightarrow -\|\underline{u}\| \|\underline{v}\| \le \underline{u} \cdot \underline{v} \le \|\underline{u}\| \|\underline{v}\|$

$\Rightarrow -15 \le \underline{u} \cdot \underline{v} \le 15$ $\quad$ min $\underline{u} \cdot \underline{v} = -15$

max $\underline{u} \cdot \underline{v} = +15$

HW2 p5

Find x_1, x_2, x_3 such that

$$x_1 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + x_2 \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} + x_3 \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_1 + 4x_2 + 7x_3 = 0$$
$$2x_1 + 5x_2 + 8x_3 = 0$$
$$3x_1 + 6x_2 + 9x_3 = 0$$

$$x_1 = -4x_2 - 7x_3$$
$$2(-4x_2 - 7x_3) + 5x_2 + 8x_3 = 0 \Rightarrow -3x_2 - 6x_3 = 0$$
$$\Rightarrow x_2 = -2x_3$$

$$3(-4(-2x_3 - 7x_3) + 5(-2x_3) + 8x_3 = 0$$

$\Rightarrow 0 = 0 \Rightarrow$ <u>any</u> value of x_3 works.

Set $x_3 = 1$, then $x_2 = -2$, $x_1 = 1$

Note: Any multiples of this also work,
e.g., $x_1 = 2$, $x_2 = -4$, $x_3 = 2$
or $x_1 = 1/2$, $x_2 = -1$, $x_3 = 1/2$ etc.

HW2 P6

a.)

	$\underline{A}$	$\underline{B}$	$\underline{C}$	$\underline{D}$	$\underline{E}$	$\underline{F}$	$\underline{G}$
Dim	3×2	3×3	3×1	2×4	3×3	2×3	1×3

b.) Square: B, E
Column: C
Row: G

c.) $\det(B) = 34$

d.) $\operatorname{tr}(\underline{E}) = 9$

e.) $a_{12} = 7$
$b_{23} = 7$
$d_{32} =$ not defined
$e_{22} = 2$
$f_{12} = 0$
$g_{12} = 6$

f.) i.) $$\underline{E} + \underline{B} = \begin{bmatrix} 5 & 8 & 15 \\ 8 & 4 & 10 \\ 6 & 0 & 10 \end{bmatrix}$$

ii.) $\underline{A} + \underline{F} =$ Not possible

iii.) $$\underline{B} - \underline{E} = \begin{bmatrix} 3 & -2 & -1 \\ -6 & 0 & 4 \\ -2 & 0 & -2 \end{bmatrix}$$

iv) $\underline{A} \times \underline{B}$ = Not possible

v) $\underline{B} \times \underline{A} = \begin{bmatrix} 54 & 76 \\ 41 & 53 \\ 28 & 39 \end{bmatrix}$

vi) $\underline{D}^T = \begin{bmatrix} 9 & 2 \\ 4 & -1 \\ 3 & 7 \\ -6 & 5 \end{bmatrix}$

vii) $\underline{E} \times \underline{B} = \begin{bmatrix} 25 & 13 & 74 \\ 36 & 25 & 75 \\ 28 & 12 & 52 \end{bmatrix}$

viii) $\underline{C}^T = \begin{bmatrix} 3 & 6 & 1 \end{bmatrix}$

ix) $\underline{A} \times \underline{C}$ = Not possible

x) $\underline{I} \times \underline{B} = \underline{B} = \begin{bmatrix} 4 & 3 & 7 \\ 1 & 2 & 7 \\ 2 & 0 & 4 \end{bmatrix}$

xi) $\underline{E}^T \times \underline{E} = \begin{bmatrix} 66 & 19 & 53 \\ 19 & 29 & 46 \\ 53 & 46 & 109 \end{bmatrix}$

xii) $\underline{C}^T \times \underline{C} = 46$

HW 2 P7

Need to check if $\underline{A}\,\underline{B} = \underline{B}\,\underline{A} = \underline{I}$ or if $\underline{A}\,\underline{C} = \underline{C}\,\underline{A} = \underline{I}$

$$\underline{A}\,\underline{B} = \begin{bmatrix} 171 & 0 & 0 & 0 \\ 68 & 1 & 0 & 0 \\ 204 & 0 & 1 & 0 \\ 68 & 0 & 0 & 1 \end{bmatrix}$$

$$\underline{B}\,\underline{A} = \begin{bmatrix} 171 & 204 & 204 & 272 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Only one is needed to show $\underline{B} \neq \underline{A}^{-1}$

$$\underline{C}\,\underline{A} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\underline{A}\,\underline{C} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

need to show <u>both</u>

$\Rightarrow \underline{C} = \underline{A}^{-1}$

HW2 P8

$\underline{A} = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$ $\underline{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ $\underline{b} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$

a.) $\underline{A}\,\underline{x} = \underline{b} \Rightarrow$ $x_1 + 2x_2 = b_1$
$-2x_1 + x_2 = b_2$

b.) $x_1 + 2x_2 = 1 \Rightarrow x_1 = 1 - 2x_2$
$-2x_1 + x_2 = 0 \Rightarrow -2(1 - 2x_2) + x_2 = 0$
$\Rightarrow -2 + 4x_2 + x_2 = 0$
$\Rightarrow 5x_2 = 2 \Rightarrow x_2 = 2/5$
$\Rightarrow x_1 = 1/5$

$\begin{bmatrix} 1/5 \\ 2/5 \end{bmatrix}$

c.) $x_1 + 2x_2 = 0 \Rightarrow x_1 = -2x_2$
$-2x_1 + x_2 = 1 \Rightarrow 4x_2 + x_2 = 1 \Rightarrow x_2 = 1/5$
$\Rightarrow x_1 = -2/5$

$\begin{bmatrix} -2/5 \\ 1/5 \end{bmatrix}$

d.) $\underline{B} = \begin{bmatrix} 1/5 & -2/5 \\ 2/5 & 1/5 \end{bmatrix}$

$\underline{A}\,\underline{B} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\underline{B}\,\underline{A} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \underline{B} = \underline{A}^{-1}$