MTH 324, Homework #5, Fall 2022 Name ______________________________________

Instructions: Show work or attach R code used to perform calculations (or any other technology used). Be sure to answer all parts of each problem as completely as possible, and attach work to this cover sheet with a staple.

1. For each of the following assertions, state whether it is a legitimate statistical hypothesis and why.
   a. $H: \sigma > 100$ b. $H: \tilde{x} = 45$ c. $H: s \leq 0.20$ d. $H: \frac{\sigma_1}{\sigma_2} < 1$ e. $H: \bar{X} - \bar{Y} = 5$
   f. $H: \lambda \leq 0.01$, where λ is the parameter of an exponential distribution used to model component failure.

2. For the following pairs of assertions, indicate which do not comply with our rules for setting up hypotheses and why.
   a. $H_0: \mu = 100, H_a: \mu > 100$
   b. $H_0: \sigma = 20, H_a: \sigma \leq 20$
   c. $H_0: p \neq 0.25, H_a: p = 0.25$
   d. $H_0: \mu_1 - \mu_2 = 25, H_a: \mu_1 - \mu_2 > 100$
   e. $H_0: S_1^2 = S_2^2, H_a: S_1^2 \neq S_2^2$
   f. $H_0: \mu = 120, H_a: \mu = 150$
   g. $H_0: \frac{\sigma_2}{\sigma_1} = 1, H_a: \frac{\sigma_2}{\sigma_1} \neq 1$
   h. $H_0: p_1 - p_2 = -0.1, H_a: p_1 - p_2 < -0.1$

3. Water samples are taken from water used for cooling as it is being discharged from a power plant into a river. It has been determined that as long as the mean temperature of the discharged water is at most $150°F$, there will be no negative effects on the river's ecosystem. To investigate whether the plant is in compliance with regulations that prohibit a mean discharge of water temperature above 150°F, 50 water samples will be taken at randomly selected times and the temperature of each sample recorded. The resulting data will be used to test the hypotheses $H_0: \mu = 150, H_a: \mu > 150$. In the context of this situation, describe Type I and Type II error. Which type of error would you consider more serious? Explain.

4. Let the test statistic Z have a standard normal distribution when H_0 is true. Give the significance level for each of the following situations. Do the same for the T test statistic with the stated degrees of freedom.
   a. $H_a: \mu > \mu_0$, rejection region $z \geq 1.88$
   b. $H_a: \mu < \mu_0$, rejection region $z \geq -2.75$
   c. $H_a: \mu \neq \mu_0$, rejection region $z \leq -2.88, or\ z \geq 2.88$
   d. $H_a: \mu > \mu_0$, df=15, rejection region $t \geq 3.733$
   e. $H_a: \mu < \mu_0$, df=24, rejection region $t \geq -2.5$
   f. $H_a: \mu \neq \mu_0$, *n*=31, rejection region $t \geq 1.697\ or\ t \leq -1.697$

5. The desired percentage of SiO_2 in a certain type of aluminous cement is 5.5. To test whether the true average percentage is 5.5 for a particular production facility, 16 independently obtained samples are analyzed. Suppose that the percentage of SiO_2 in a sample is normally distributed with $\sigma = 0.3$, and $\bar{x} = 5.25$.

a. Does this indicate conclusively that the true average percentage differs from 5.5? Conduct an appropriate hypothesis test.
b. If the true average percentage is $\mu = 5.6$, conduct an appropriate hypothesis test using a level of $\alpha = 0.01$ with $n = 16$.
c. What value of n is required to satisfy $\alpha = 0.01$ and $\beta(5.6) = 0.01$?

6. A manufacturer of nickel-hydrogen batteries randomly selects 100 nickel plates for test cells, cycles them a specified number of times, and determined that 14 of the plates have blisters.
   a. Does this provide compelling evidence for concluding that more than 10% of all plates blister under such circumstances? State and test the appropriate hypotheses using a significance level of 0.05. In reaching your conclusion, what type of error might you have committed?
   b. Is it is really the case that 15% of all plates blister under these circumstances and a sample size of 100 is used, how likely is it that the null hypothesis of part (a) will not be rejected by the level 0.05 test? What if the same size is 200?

7. For which P-values would the null hypothesis be rejected when performing a level 0.05 test?
   a. 0.001   b. 0.21   c. 0.078   d. 0.047   e. 0.148

8. Pairs of P-values and significance levels, α are given. For each pair, state whether the observed P-value would lead to rejection of H_0 at the given significance level.
   a. P-value = 0.084, $\alpha = 0.05$
   b. P-value = 0.003, $\alpha = 0.001$
   c. P-value = 0.498, $\alpha = 0.05$
   d. P-value = 0.084, $\alpha = 0.10$
   e. P-value = 0.039, $\alpha = 0.01$
   f. P-value = 0.218, $\alpha = 0.10$

9. Give as much information as you can about the P-value of a t-test in each of the following situations.
   a. Upper-tailed, df=8, t=2.0
   b. Lower-tailed, df=11, t = -2.4
   c. Two-tailed, df=15, t = -1.6
   d. Upper-tailed, df=19, t = -0.4
   e. Lower-tailed, df=5, t=5.0
   f. Two-tailed, df=40, t = -4.8

10. Describe the difference between statistical and practical significance.

11. Tensile strength tests were carried out on two different grades of wire rod, resulting in the accompanying data.

Grade	**Sample Size**	**Sample Mean** $\left(\frac{kg}{mm^2}\right)$	**Sample Standard Deviation**
AISI 1064	$m = 129$	$\bar{x} = 107.6$	$s_1 = 1.3$
AISI 1078	$n = 129$	$\bar{y} = 123.6$	$s_2 = 2.0$

a. Does the data provide compelling evidence for concluding that true average strength for the 1078 grade exceeds that for the 1064 grade by more than 10 $\frac{kg}{mm^2}$? Test the appropriate hypotheses using the P-value approach.
b. Estimate the difference between true average strengths for the two grades in a way that provides information about precision and reliability.

12. Low back pain (LBP) is a serious health problem in many industrial settings. An article from 1995 reported the accompanying summary data on lateral range of motion (degrees) for a sample of workers without a history of LBP and another sample with a history of this malady.

Condition	Sample Size	Sample Mean	Sample Standard Deviation
No LBP	28	91.5	5.5
LBP	31	88.3	7.8

Calculate a 90% confidence interval for the difference between population mean extent of lateral motion for the two conditions. Does the interval suggest that population mean lateral motion for the two conditions differs? Is the message different is a confidence level of 95% is used?

13. Scientists and engineers frequently wish to compare two different techniques for measuring or determining the value of a variable. In such situations, interest centers on testing whether the mean difference in measurements is zero. An article reports the accompanying data on amount of milk ingested by each of 14 randomly selected infants.

	1	**2**	**3**	**4**	**5**	**6**	**7**
DD Method	1509	1418	1561	1556	2169	1760	1098
TW Method	1498	1254	1336	1565	2000	1318	1410
Difference	11	164	225	-9	169	442	-312
	8	**9**	**10**	**11**	**12**	**13**	**14**
DD Method	1198	1479	1281	1414	1954	2174	2058
TW Method	1129	1342	1124	1468	1604	1722	1518
Difference	69	137	157	-54	350	452	540

a. Is it plausible that the population of differences is normal?
b. Does it appear that the true average difference between intake values measured by the two methods is something other than zero? Determine the P-value of the test, and use it to reach a conclusion at significance level 0.05.

14. Researchers sent 5000 resumes in response to job ads that appeared in the *Boston Globe* and *Chicago Tribune*. The resumes were identical except that 2500 of them had “white” sounding names like Brett and Emily, and 2500 had “black” sounding names like Tamika and Rasheed. The resumes of the first type elicited 250 responses and the resumes of the second type only 167 responses. Does this data strongly suggest that a resume with a “black” name is less likely to result in a response than is a resume with a “white” name?