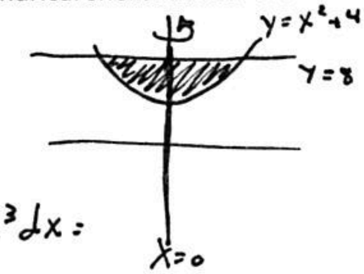
MTH 264, Quiz #2, Fall 2022 Name KEY

Instructions: Work the problems below as directed. Show all work. Clearly mark your final answers. Use exact values unless the problem specifically directs you to round. Simplify as much as possible. Partial credit is possible, but solutions without work will not receive full credit.

1. Find the volume of the solid formed from revolving the region bounded by $y = x^2 + 4, y = 8, x = 0$ around the y-axis using the method of cylindrical shells. Sketch the region.



$x^2+4=8$
$x^2-4=0$
$x=\pm 2$

$2\pi\int_0^2 x\,(8-(x^2+4))\,dx =$

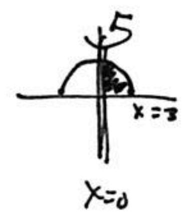
$2\pi\int_0^2 x(4-x^2)\,dx = 2\pi\int_0^2 4x - x^3\,dx =$

$2\pi\left[2x^2 - \frac{1}{4}x^4\right|_0^2 = 2\pi[\,4\,] = \boxed{8\pi}$

2. Find the surface area of the hemisphere found by revolving $y = \sqrt{9 - x^2}$ around the y-axis on the interval [0,3].

$y' = \frac{1}{2}(9-x^2)^{-1/2}\cdot(-2x) = -\frac{x}{\sqrt{9-x^2}}$

$2\pi\int_0^3 x\sqrt{1+\left(\frac{-x}{\sqrt{9-x^2}}\right)^2}\,dx = 2\pi\int_0^3 x\sqrt{1+\frac{x^2}{9-x^2}}\,dx$



$= 2\pi\int_0^3 x\sqrt{\frac{9-\cancel{x^2}+\cancel{x^2}}{9-x^2}}\,dx = 2\pi\int_0^3 x\sqrt{\frac{9}{9-x^2}}\,dx =$

$2\pi\int_0^3 \frac{3x}{\sqrt{9-x^2}}\,dx$ $\quad u = 9-x^2$, $du = -2x\,dx$, $-\frac{1}{2}du = x\,dx$ $\quad \int -\frac{3}{2}u^{-1/2}\,du = -\frac{3}{\cancel{2}}\cdot u^{1/2}\cdot \cancel{2} \to -3\sqrt{9-x^2}\Big|_0^3 =$

$3(3)=9$

$2\pi(9) = \boxed{18\pi}$

3. Find the arc length of the function $y = x^2 + x - 2$ on the interval [-2,1].

$y' = 2x+1$

$\int_{-2}^1 \sqrt{1+(2x+1)^2}\,dx = \int_{-2}^1 \sqrt{1+4x^2+4x+1}\,dx = \int_{-2}^1 \sqrt{4x^2+4x+2}\,dx$

$= \frac{\ln(6\sqrt{10}+19)}{4} + \frac{3\sqrt{10}}{2} \approx 5.65264$